Personalized Pricing with Consumers' Quality Uncertainty

Guangzhi Chen
University of Florida, guangzhi.chen@warrington.ufl.edu

Tianxin Zou
University of Florida, tianxin.zou@warrington.ufl.edu

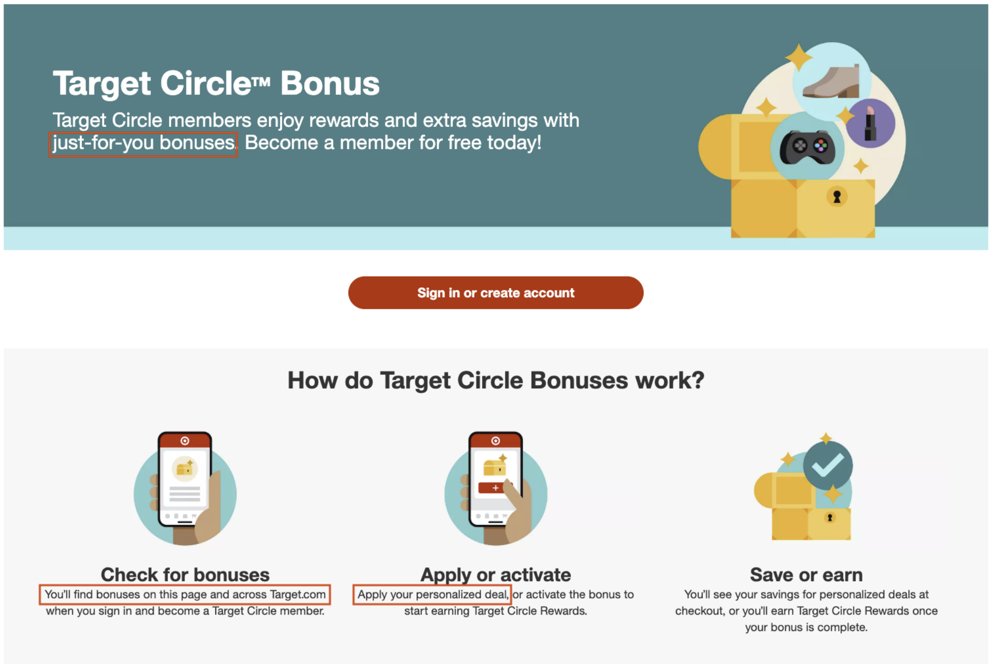
Abstract: We examine a firm's personalized pricing (PP) strategies in markets where consumers are uncertain about product quality. In such markets, prices serve not only as a tool for price discrimination but also as a means of conveying quality information to consumers. We reveal that a firm faces a tradeoff between adopting PP to better price discriminate among consumers and not adopting it to signal its high quality. We find that a high-quality firm should adopt PP only when its product quality is known to either a very small or a very large fraction of consumers, and when its high-quality product, on average, offers either very low or very high additional value to consumers relative to a low-quality product. Moreover, the high-quality firm may charge consumers personalized prices less than their willingness to pay to signal its quality in equilibrium, deviating from first-degree price discrimination when consumers are informed about quality. Counterintuitively, when more consumers know product quality or when the high-quality product provides higher average value to consumers, consumer surplus and social welfare may decrease, but a low-quality firm's profit may increase. Furthermore, the firm's profit can be lower when its personalized pricing leverages more information about consumer characteristics. Randomized experiments provide evidence that a personalized price is a weaker signal of objective product quality than a uniform price.

Key words: Personalized Pricing, Quality Signaling, Consumer Uncertainty, Personalization, Price Discrimination

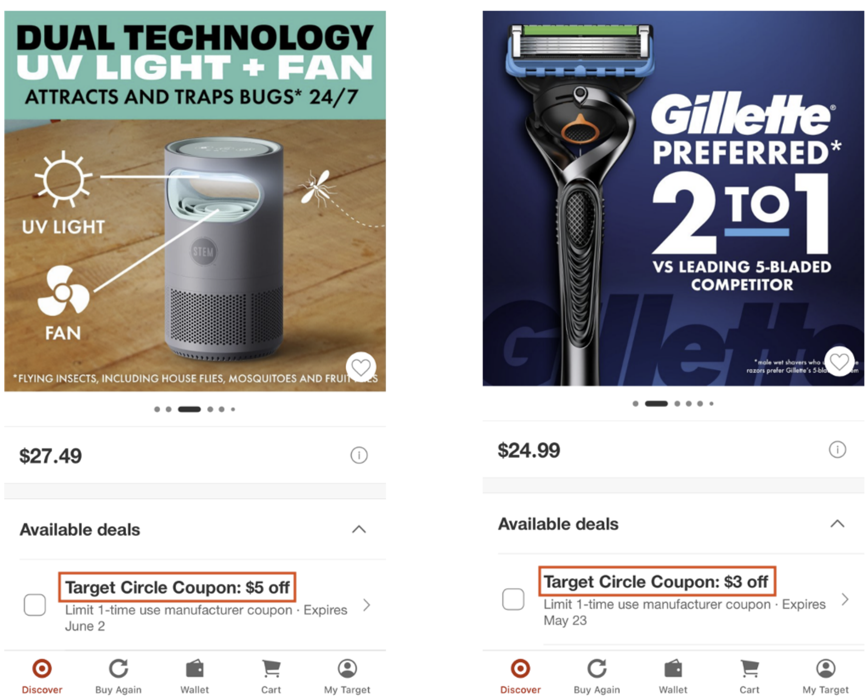
1 Introduction

An increasing number of firms adopt personalized pricing (PP), leveraging consumer data to tailor prices for individual consumers. Many firms across various industries, including Discover Financial Services, Progressive Insurance, Home Depot, JCPenney, Staples, Kroger, Safeway, McDonald's, Carlsberg, Adidas, Reebok, Eddie Bauer, and River Island, have practiced personalized pricing (Valentino-DeVries et al. 2012, Banjo 2014, Skrovan 2017, Kelso 2022, Progressive 2024, Talon.One 2025). In practice, PP is often implemented in the form of personalized coupons or discounts communicated to each consumer via private channels, such as personal web or app accounts, emails, and text messages. Figure 1 presents examples of personalized coupons delivered to consumers through their individual user accounts. Firms either develop their own digital PP systems or adopt third-party ready-to-deploy PP solutions, such as Amazon Personalize, Vouchery, and Talon.One. For instance, Talon.One has supported firms including Adidas, Reebok, Eddie Bauer, and River Island in generating personalized coupons. Recent empirical research has shown that personalized pricing can significantly improve firms' profitability (e.g., Dubé and Misra 2023, Liu 2023).

Figure 1. Examples of personalized coupons communicated through individual user accounts



(a) Target's description of its personalized deals



(b) Personalized coupons in the Target app

It is well established that PP enables firms to engage in (first-degree) price discrimination by charging consumers' willingness to pay (WTP) when consumers have perfect knowledge about product quality (Pigou 1920). However, in many market situations—particularly for experience products that are newly introduced to the market, and for which limited quality information is publicly available—consumers often face uncertainty about product quality at the time of purchase. Their uncertainty about quality significantly complicates the firm's PP strategy, as simply setting the price equal to consumers' WTP becomes invalid for two important reasons. First, because consumers are uncertain about product quality, they do not know their true WTP for the product. Second, consumers with quality uncertainty can infer product quality from the prices they receive—prices typically serve as signals of product quality—so the personalized prices can endogenously affect consumers' WTP for the product. While this signaling role of prices has been extensively studied in the context of uniform pricing (e.g., Milgrom and Roberts 1986, Bagwell and Riordan 1991, Desai and Srinivasan 1995, Soberman 2003, Chen et al. 2024), the extant literature has yet to explore how prices can still signal product quality when these prices can be personalized.

Recent academic research and industry reports suggest that personalized pricing can alter how consumers infer product quality from prices, often attenuating the effectiveness of price as a quality signal. For example, Spann et al. (2024) argue that when personalized pricing algorithms generate substantial price variation across consumers, individuals become less likely to treat the prices they receive as reliable quality signals, as the informational role of price is thereby obscured. Similarly, Zagorsky (2025) notes, "In a custom-pricing situation, seeing a high price doesn't mean something is higher quality. Instead, a high price simply means a business views the customer as willing to part with more money." We also observe that firms using personalized pricing attempt to reassure consumers about product quality. For example, Kroger emphasizes that "customers never have to compromise *high-quality* for low prices" while promoting its *personalized* savings features.[1]

To provide further evidence that personalized pricing can influence consumers' quality inference, we also conducted four randomized online experiments in different product contexts, where consumers were asked to evaluate a new product's quality based on the prices (personalized vs. uniform) they received. These experiments consistently show that personalized pricing, compared to uniform pricing, significantly weakens consumers' inference of product quality from price. Moreover, some participants explicitly articulated that personalized pricing undermined their tendency to interpret a high price as an indicator of high quality. For example, one participant noted, "I am thinking that the higher price would likely mean higher quality but my confidence is decreased by the personalized price." Another said, "They are probably charging more because of the personalization aspect. I doubt they are any better than the average priced pair." Note that this mechanism is distinct from previously documented downsides of personalized pricing, such as fairness

[1] See https://ir.kroger.com/news/news-details/2026/Kroger-Unveils-Customers-2025-Yearly-Checkout/.

concerns (Allender et al. 2021, Cohen et al. 2022) or the erosion of luxury brand exclusivity. Importantly, our experiments measured consumers' perceptions of *objective product quality*, whereas the existing literature typically studies factors that influence willingness to pay but are *independent of consumers' objective quality perceptions*.

The preceding evidence indicates that when consumers are uncertain about product quality, a firm's PP strategy must account not only for price discrimination but also for its influence on consumers' quality inference. This paper examines how a firm should optimally design its PP strategy in markets where consumers have imperfect knowledge of product quality. Specifically, we address the following questions: (1) How should a firm design its PP strategy? Specifically, when should the firm adopt personalized pricing, and if so, how should it set personalized prices for different consumers? (2) How will the firm's PP strategy affect its profit and consumer welfare? (3) Does having more information about consumers increase the firm's benefit from PP?

We develop an analytical model where a monopoly firm sells a product whose quality can be either high or low. Consumers differ in their valuations and informedness about product quality. In particular, quality-sensitive consumers appreciate the premium features offered by a high-quality product and thus are willing to pay more for it than a low-quality product, whereas quality-insensitive consumers do not value these premium features and their willingness to pay is independent of product quality. In addition, informed consumers know about the product quality, whereas uninformed consumers do not. The firm decides whether to adopt the PP technology at an adoption cost, which allows it to personalize prices based on consumers' quality sensitivity and informedness. If the firm adopts the PP technology, consumers will observe only their own personalized prices but not the prices for other consumers. As noted, this assumption reflects that in most digital PP practices, consumers receive personalized price offerings via private communications. By contrast, if the firm does not adopt the PP technology, it charges a uniform price for all consumers. An uninformed consumer updates her belief about the firm's product quality based on what she observes about the firm's pricing decisions, including whether it adopts the PP technology and the corresponding personalized or uniform price. Thus, the firm has incentives to influence uninformed quality-sensitive consumers' quality beliefs via its pricing decisions.

Our first finding is that with consumers' quality uncertainty, a high-quality firm has incentives to strategically refrain from adopting the PP technology. This result arises because the adoption of PP, as compared to charging a uniform price, may weaken a high-quality firm's effectiveness in signaling its high quality. As established in Bagwell and Riordan (1991), in the context of uniform pricing, a high-quality firm can signal its high quality with a high uniform price, because at this high price, quality-sensitive consumers will not buy the product if they believe or know that the product has low quality. A firm whose product quality is indeed low would find it unprofitable to mimic the high price because doing so would forgo informed quality-sensitive consumers; it would prefer to capture these consumers as well as quality-insensitive consumers with a low uniform price. Consequently, uninformed quality-sensitive consumers rationally infer high quality from the high uniform price. By contrast, when the high-quality firm adopts the PP technology, charging such a high (personalized) price to uninformed quality-sensitive consumers can no longer convince them of its high quality. This is because a low-quality firm would be strongly inclined to also adopt the PP technology and mimic the high personalized price for these consumers. The mimicry does not prevent the firm from selling to other consumers with low personalized prices, which are unobservable to uninformed quality-sensitive consumers. Therefore, in markets with consumers' quality uncertainty, the high-quality firm faces a tradeoff regarding the adoption of the PP technology: doing so allows the firm to price discriminate among consumers but may prevent it from effectively signaling its high quality. The finding is consistent with and provides a micro-foundation for the empirical observations discussed above.

In light of this tradeoff, how should a firm design its PP strategy in a market with consumers' quality uncertainty? Specifically, when should it adopt the PP technology, and if so, how should it personalize prices for different consumers? Our second main result reveals that the prevalence of consumer quality uncertainty in the market has a non-monotonic impact on the adoption of the PP technology by a high-quality firm: it should adopt the PP technology either when its quality is known to most consumers or when it is known to few consumers. When most consumers are already informed about product quality, a firm with low product quality has very weak incentives to mimic the high-quality firm. In this case, if the high-quality firm adopts the PP technology, it can secure substantial profit from extracting the surplus of most quality-sensitive consumers who are informed about product quality. Conversely, when most consumers are uninformed about quality, the low-quality firm has very strong mimicry incentives. In this case, even if the high-quality firm does not adopt the PP technology, it still needs to charge a low uniform price to discourage the low-quality firm's price mimicry. In these two cases, the high-quality firm will adopt the PP technology to benefit from price discrimination. Otherwise, the firm should refrain from adoption to benefit from effectively signaling its high quality. In addition, we find that the high-quality firm should refrain from adopting the PP technology when the size of quality-sensitive consumers is intermediate and when their appreciation for the high-quality features is moderate; otherwise, the firm should adopt the PP technology. In sharp contrast, we find that consumers' quality uncertainty can facilitate a low-quality firm's adoption of the PP technology, provided that the high-quality firm also adopts it. In this case, the low-quality firm's adoption of the PP technology allows it to mimic the high-quality firm's personalized prices and hence profit more from uninformed quality-sensitive consumers.

We also find that when the high-quality firm adopts the PP technology, it may charge uninformed quality-sensitive consumers a personalized price lower than their WTP for its product, departing from the classic first-degree price discrimination result. This arises either in equilibria where the lowered price would make the low-quality firm unprofitable to mimic the high-quality firm's price for these consumers, or in equilibria where the low-quality firm successfully mimics this price so that these consumers' willingness to pay is based on their prior quality belief. Interestingly, in the latter case, the low-quality firm can charge uninformed quality-sensitive consumers a personalized price higher than their WTP for a low-quality product.

Our third main finding is that the firm's strategic decisions on its PP strategy can lead to unintended market consequences. First, we find that when more consumers become informed about product quality, consumer surplus and social welfare may decrease, whereas the low-quality firm's profit may increase. A practical implication is that, in markets where consumers infer quality from price, greater transparency about product quality, facilitated by increasingly popular review systems and influencer channels, can harm consumers and society by shifting the firm's equilibrium PP strategy. As explained, when most consumers are informed about product quality, in equilibrium, the high-quality firm may adopt the PP technology and lower the personalized price for uninformed quality-sensitive consumers to make mimicry unprofitable for the low-quality firm. When the number of informed consumers further increases, the low-quality firm's mimicking incentives weaken, so the high-quality firm can lower its price to a lesser extent, thereby reducing the total consumer surplus. By contrast, when the number of informed consumers increases from a medium level to a high level, the high-quality firm switches from not adopting the PP technology to adopting it to focus on price discrimination while tolerating the low-quality firm's mimicry of the personalized price for uninformed quality-sensitive consumers. This potentially increases the low-quality firm's total profit. In a similar vein, we find that consumer surplus and social welfare may decrease when the high-quality product generates more value to consumers on average relative to the low-quality one, and when implementing personalized pricing becomes less costly.

We further demonstrate that the above findings are qualitatively robust when the firm observes only consumers' valuation but not their informedness about product quality. Interestingly, the high-quality firm's equilibrium profit can be higher when it does not observe consumer informedness than when it does. This suggests that more knowledge about consumers can backfire on a high-quality firm when some consumers are uncertain about product quality. Our results are also robust when the firm can accompany its personalized prices with a list price, when some consumers do not observe the firm's adoption of personalized pricing, when quality-insensitive consumers also have a higher willingness to pay for the high-quality product, when the high-quality firm incurs a higher marginal cost, and when the firm's cost of adopting personalized pricing is its private information.

In summary, this research offers new insights into a firm's personalized pricing strategy in markets where prices serve as quality signals and consumers observe only their own personalized price. Our findings show that, in such markets, a high-quality firm must carefully balance the dual roles of personalized pricing as a tool for surplus extraction and a device for quality signaling. We find that the high-quality firm optimally refrains from adopting personalized pricing when the product's premium features are only moderately valued by consumers, or when an intermediate share of consumers is informed of product quality. Such conditions arise naturally, for instance, when the product has been on the market long enough for quality information to reach some consumers through reviews, word-of-mouth, or advertising, but not yet the majority. They also arise when the product imposes moderate requirements, such as background knowledge or technical expertise, for consumers to access and comprehend quality information. We further find that, in the markets we study, a high-quality firm should be careful about conditioning its prices on consumer informedness (for instance, enabled by collecting additional consumer data or deploying more advanced artificial intelligence algorithms), because doing so can weaken its quality signaling and reduce its profit. Lastly, we find that several seemingly beneficial market developments can have counterproductive effects on consumer and social welfare, because they reshape the firm's quality signaling incentives and shift its pricing decisions. Policymakers and consumer advocates should therefore account for the firm's strategic response when promoting greater accessibility of quality information (e.g., through expanded consumer review platforms or stricter disclosure regulations), improvements in product quality (e.g., through advances in materials or quality control), and reductions in the cost of implementing personalized pricing (e.g., driven by pre-trained artificial intelligence models or lower data acquisition costs).

The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3 describes the model setup, and Section 4 presents the equilibrium analysis. Section 5 extends the analysis to situations where the firm observes only partial consumer information, where the firm can accompany its personalized prices with a list price, and where the firm can also signal through advertising. Section 6 presents our randomized online experiments. Section 7 concludes the paper.

2 Literature

This research is related to the growing literature on personalized pricing. Pigou (1920) suggests that personalized pricing allows a firm to extract more surplus from consumers based on their heterogeneous willingness to pay. Recent empirical research has studied how to quantify the value of personalized pricing over uniform pricing, how to implement personalized pricing in practice, and its welfare impacts (Elmachtoub et al. 2021, Dubé and Misra 2023, Liu 2023).

A large body of theoretical literature demonstrates the strategic effects of personalized pricing, including how personalized pricing affects price competition (Thisse and Vives 1988, Chen et al. 2001, Shaffer and Zhang 1995, 2002, Bhaskar and To 2004, Liu and Serfes 2004, Choudhary et al. 2005, Ghose and Huang 2009, Chen et al. 2017, Rhodes and Zhou 2024, Li et al. 2024), influences the interaction between channel members (Liu and Zhang 2006, Jullien et al. 2023), triggers consumers' fairness concerns (Allender et al. 2021, Cohen et al. 2022), incentivizes consumers to avoid or manipulate data tracking and protect privacy (Taylor 2004, Acquisti and Varian 2005, Chen et al. 2020, Li and Li 2023, Anderson et al. 2023), shapes consumers' beliefs about how many other consumers will buy a product with network effects (Hajihashemi et al. 2022), and alters consumers' incentives to search for product valuations (Wang et al. 2023). Several papers study how a firm's personalized pricing strategy interacts with the firm's other strategic decisions, such as consumer addressability (Chen and Iyer 2002), personalized advertising (Anderson et al. 2015), product personalization (Du and Ning 2025), and consumer information sharing (Hu et al. 2025).

The aforementioned papers assume that product quality is common knowledge. By contrast, our paper studies how a firm should design its PP strategy when consumers have quality uncertainty, which is prevalent in markets for experience products and new products. In such contexts, pricing serves not only the role of extracting consumer surplus but also the role of conveying quality information. We show that a firm may find it profitable to refrain from adopting personalized pricing, forgoing price discrimination in favor of quality signaling. Therefore, our paper contributes to the literature by documenting the strategic effect of personalized pricing on shaping consumers' beliefs about product quality. In addition, it provides a microfoundation for, and is consistent with, the experimental and empirical evidence that personalized pricing weakens consumers' tendency to infer product quality from observed prices.

To improve the quality of decision making, a firm often seeks to acquire more information about consumers (Bergemann et al. 2018). When consumers know product quality, the earlier research indicates that a monopolistic firm can generally receive more profit from personalized pricing if it has more information about its consumers (Armstrong 2006). By contrast, in the presence of quality uncertainty, we show that a monopolist's profit can decrease when the firm can personalize prices based on more granular consumer information. Our paper provides a potential explanation for why a firm may strategically adopt a pricing algorithm with less granular targeting.

Our study is also related to the literature on the quality signaling role of pricing (Milgrom and Roberts 1986, Bagwell and Riordan 1991, Desai and Srinivasan 1995, Stiving 2000, Zhao 2000, Soberman 2003, Janssen and Roy 2010, Guo and Jiang 2016, Guo and Wu 2016, Jiang and Yang 2019, Kuksov and Liao 2019, Chen and Jiang 2021, Bolandifar et al. 2023, Chen et al. 2024, 2025). Most of these papers illustrate how a firm can use its uniform price, sometimes together with other instruments (e.g., advertising, money-back guarantee), to signal its high quality. By contrast, we consider the firm's quality signaling problem when prices can be personalized. We show that when a high-quality firm adopts personalized pricing, it can choose to *lower* its personalized prices from the perfect-information level to signal high quality. This is in sharp contrast to the context of uniform pricing, where the high-quality firm raises its price to signal high quality.

In particular, two papers have studied a firm's signaling problem when the firm can price discriminate consumers who are uncertain about product quality or valuation. Anderson and Simester (2001) show that implementing price discrimination can be an adverse signal of product quality. This result relies on the fact that consumers observe the discriminatory prices for all other consumers, and that the marginal cost of a high-quality firm is higher than the willingness to pay of low-valuation consumers. Because serving low-valuation consumers is unprofitable for a high-quality firm, consumers will infer low quality if they observe the firm serving these consumers with a low discriminatory price. The mechanism described in

Anderson and Simester applies *only* to traditional, non-digital price discrimination (e.g., student discounts), where prices of all consumers are publicly observable. In stark contrast, our mechanism uniquely applies to most modern practices of digital personalized pricing, where prices are privately communicated and consumers observe only their own personalized price.

Xu and Dukes (2022) consider a situation where consumers are uncertain about their product valuations but receive signals that are either positively or negatively biased, making consumers potentially overestimate or underestimate their valuations, respectively. The firm personalizes prices and has private information about whether consumers tend to underestimate or overestimate their valuations. They show that a firm can signal to consumers that they tend to underestimate their valuations by accompanying its personalized prices with a list price. This is because in their setting, the firm tends to charge lower prices when consumers underestimate their valuations, so a list price, which informs consumers about the market price ceiling, can convince consumers that the personalized prices of other consumers cannot be too high. In sharp contrast, our paper shows that when consumers are uncertain about product quality, a high-quality firm cannot signal its quality by accompanying its personalized prices with a list price. This is because in our setting, the type of firm that wants to have its type revealed (the high-quality firm) tends to charge higher personalized prices, but using a list price suggests that prices tend to be low.

3 Model

3.1 Market basics

A monopoly firm sells a product, whose quality is either high (h) or low (l) and is the firm's private information. We refer to $j \in \{h, l\}$ as the firm's quality type. Let $\lambda \in (0, 1)$ be the prior probability of high quality $j = h$. A low-quality product provides essential functionalities satisfying consumers' basic needs, and a high-quality product additionally includes premium features that potentially enhance their experience. In the main model, the marginal cost is normalized to zero regardless of the firm's quality, which helps us rule out cost-driven mechanisms on how the adoption of personalized pricing affects a firm's quality signaling in the literature (e.g., Anderson and Simester 2001). In an extension, we show that our qualitative insights remain unchanged even if the high-quality firm incurs a higher cost than the low-quality firm, as long as the cost is not prohibitively high.

There is a unit mass of heterogeneous consumers. They differ in their informedness about product quality (Bagwell and Riordan 1991). *Informed* consumers (denoted as $\tau_i = I$), whose size is $0 < \alpha < 1$, know product quality. By contrast, *uninformed* consumers (denoted as $\tau_i = U$), whose size is $1 - \alpha$, do not know product quality but only its prior distribution. In reality, the parameter α tends to be low when a product is newly introduced to the market such that little product quality information (e.g., product reviews, consumer word-of-mouth) is available, or when there are substantial requirements (e.g., education level, domain knowledge) for consumers to access and comprehend the product quality information.

Consumers also differ in their *sensitivity* to product quality (Kirmani and Rao 2000). *Quality-sensitive* consumers (denoted as $\theta_i = S$), whose size is $0 < \beta < 1$, appreciate the premium features offered by the high-quality product: their willingness to pay (WTP) for the low-quality product is normalized to 1 without loss of generality, and their willingness to pay for the high-quality product is $1 + d$, where $d > 0$ captures the additional value derived by quality-sensitive consumers from the high-quality product. By contrast, *quality-insensitive* consumers (denoted as $\theta_i = N$), whose size is $1 - \beta$, are relatively insensitive to product quality. In the main model, to simplify the exposition, we assume that they do not value the premium features of the high-quality product and are already satisfied with the basic functionalities: their willingness to pay is 1 regardless of the product quality. For example, outdoor runners (quality-sensitive consumers) are willing to pay more for running shoes with strong waterproof and anti-slip features, but indoor runners (quality-insensitive consumers) do not value these features. Therefore, consumer i's willingness to pay for type-j firm's product is $v_{ij} = 1 + d$ if the consumer is quality-sensitive and the product has high quality, and is $v_{ij} = 1$ otherwise.[2]

In summary, our main model contains four groups of consumers, differing in their quality sensitivity θ_i (high versus low) and their quality informedness τ_i (informed versus uninformed). A consumer's type is summarized as (θ_i, τ_i). The segment size is $\alpha\beta$ for informed quality-sensitive consumers, $\alpha(1-\beta)$ for informed quality-insensitive consumers, $(1-\alpha)\beta$ for uninformed quality-sensitive consumers, and $(1-\alpha)(1-\beta)$ for uninformed quality-insensitive consumers.

3.2 Personalized pricing technology

Without the personalized pricing (PP) technology (denoted $w = 0$), the firm charges a uniform price p to all consumers. With the PP technology (denoted $w = 1$), the firm sets personalized prices $P(\theta, \tau)$ based on each consumer's type (θ, τ).[3] In practice, a consumer's type may be inferred from her demographics, geographic location, browsing and purchase history of related products, and other observables. Moreover, the firm can acquire this information even if the consumer has not previously purchased the product. The firm's action (denoted σ) therefore takes one of two forms: $(w = 0, p)$ or $(w = 1, P(\theta, \tau))$. We use $w(\sigma)$, $p(\sigma)$, and $P(\sigma, \theta, \tau)$ to denote the corresponding variables when the firm plays σ. The firm's action fully summarizes its PP strategy: whether to adopt the PP technology and, conditional on adoption, the schedule of personalized prices across consumer types. Section 5.1 establishes the robustness of our results to the case in which the firm observes θ but not τ. To adopt the PP technology, the firm incurs a fixed cost k, assumed to be common knowledge in the main analysis. This assumption reflects that consumers share a market-level

[2] In Online Appendix E, we show that our qualitative insights are robust in a more general setting where (1) quality-insensitive consumers also have a strictly higher willingness to pay for the high-quality product than for the low-quality one, and (2) the low-quality product also provides higher value to quality-sensitive consumers than to quality-insensitive ones.

[3] In principle, the firm may set different prices for consumers of the same type. However, our analysis shows that the firm optimally charges the same price to all consumers of a given type, since they are identical from the firm's perspective.

perception that personalized pricing is costly to implement. In Online Appendix H, we show that our key insight about PP adoption qualitatively remains valid under imperfect cost observability, using a binary-cost specification in which the adoption cost is the firm's private information and unknown to consumers.[4]

In practice, whether a firm employs personalized pricing is often known to its customers. Major economies such as the European Union and China require firms to explicitly disclose the use of personalized pricing based on consumer profiles.[5] Many firms also voluntarily disclose to their consumers that the coupons and discounts they receive are personalized to maintain trust and transparency, as illustrated by the examples in Figure 1. Conversely, if a firm does not adopt personalized pricing, consumers can readily verify that the price they receive matches the publicly listed price or that offered to other consumers. Therefore, in the main model, we assume that consumers can observe whether the firm adopts the PP technology (w).[6]

Additionally, in most modern, online practices of personalized pricing, firms communicate personalized prices to consumers via private channels, such as text messages, emails, or personal app accounts, to alleviate consumers' privacy and fairness concerns. Consequently, consumers usually observe only their own personalized price but not the prices for other consumers (UK Competition and Markets Authority 2018, Hajihashemi et al. 2022, Xu and Dukes 2022, Du and Ning 2025, Liu 2023). This sharply contrasts with traditional offline price-discrimination settings considered by the prior literature, for example, student discounts or installment billing options, where all price levels are publicly observable (Anderson and Simester 2001). Accordingly, we assume that if the firm adopts the PP technology, consumers observe only their own personalized price but not those received by other consumers.

Formally, let $\gamma_i(\sigma)$ denote consumer i's *observed information* of the firm's action σ. If the firm does not adopt the PP technology, the consumer observes $w = 0$ and the uniform price: $\gamma_i(\sigma) = (w(\sigma) = 0, p(\sigma))$; if the firm uses its PP technology, the consumer observes $w = 1$ and her own personalized price: $\gamma_i(\sigma) = (w(\sigma) = 1, P(\sigma, \theta_i, \tau_i))$; that is, the consumer observes only a point, $P(\sigma, \theta_i, \tau_i)$, of the firm's personalized pricing function, $P(\sigma, \theta, \tau)$.

A consumer's expected utility of purchasing the product depends on her knowledge about product quality. Since informed consumers know product quality, an informed consumer i's expected utility is

$$u(\sigma, \theta_i, \tau_i = I) = \begin{cases} v_{ij} - p(\sigma) & \text{if } w(\sigma) = 0 \\ v_{ij} - P(\sigma, \theta_i, \tau_i) & \text{if } w(\sigma) = 1. \end{cases} \tag{1}$$

[4] We acknowledge that consumers may not know the exact adoption cost in practice, and our assumption of observable signaling costs is a modeling simplification common in the signaling literature. In practice, related information may be available online: many firms rely on third-party PP solutions whose providers (e.g., Amazon Personalize, Microsoft Azure AI Personalizer, and Voucherify) publish pricing menus and cost calculators, enabling consumers to form approximate cost estimates. See `https://aws.amazon.com/personalize/pricing/`, `https://www.voucherify.io/pricing`, and `https://azure.microsoft.com/en-us/pricing/details/cognitive-services/personalizer/`.

[5] See `https://eur-lex.europa.eu/eli/dir/2019/2161/oj/eng` and `https://www.skadden.com/insights/publications/2021/11/chinas-new-data-security-and-personal-information-protection-laws`.

[6] Online Appendix D shows that our results are robust if some consumers do not observe w.

By contrast, uninformed consumers form their beliefs about product quality based on their observed information $\gamma_i(\sigma)$. In other words, our setting involves a signaling game. Because the willingness to pay of quality-insensitive consumers is independent of product quality in the main model, we can focus on discussing the quality belief of uninformed quality-sensitive consumers. We use $\phi(\gamma_i(\sigma))$ to represent their believed probability that the firm has high quality ($j = h$), based on the observed information $\gamma_i(\sigma)$. We will delineate consumers' quality beliefs in the next section. Thus, an uninformed consumer i's expected utility is

$$u(\sigma, \theta_i, \tau_i = U) = \begin{cases} E[v_{ij}] - p(\sigma) & \text{if } w(\sigma) = 0 \\ E[v_{ij}] - P(\sigma, \theta_i, \tau_i) & \text{if } w(\sigma) = 1. \end{cases} \tag{2}$$

Here, $E[v_{ij}] = 1 + \phi(\gamma_i(\sigma)) \cdot d$ if the consumer is quality-sensitive, and $E[v_{ij}] = 1$ if quality-insensitive. The consumer buys the product if $u(\sigma, \theta_i, \tau_i) \geq 0$.

We have two remarks about our model setup and assumptions. First, in the main model, when the firm uses the PP technology, it does not announce a list price. Section 5.2 relaxes this assumption and shows that our results are qualitatively unchanged. Second, even if the firm uses the PP technology ($w = 1$), it can still charge the same price for all consumers by choosing $P(\theta, \tau)$ as a constant. However, doing so wastes the adoption cost k and is hence suboptimal.

We make the following parameter assumptions throughout the analysis:

$$\beta(1 + d) > 1, \text{and } k < 1 - \beta. \tag{3}$$

The first condition suggests that the population of quality-sensitive consumers is not too low and that their valuations for the high-quality and low-quality products differ sufficiently. This allows us to focus on the interesting case where the high-quality firm has sufficient incentives to let these consumers know about its high quality. The second condition suggests that the fixed adoption cost of the PP technology is not prohibitively high. Under these two conditions, the high-quality firm will choose to adopt the PP technology in the absence of quality uncertainty, a benchmark case we will discuss in Lemma 1.[7]

3.3 Equilibrium concept

We solve for pure-strategy perfect Bayesian equilibria (PBE). In a PBE, quality-insensitive consumers will buy the product if and only if its price is no greater than one, and informed quality-sensitive consumers will buy if and only if the price is no higher than $1 + d$. By contrast, an uninformed quality-sensitive consumer

[7] When all consumers know the product quality and the high-quality firm does not adopt the PP technology, the firm can either (i) sell only to quality-sensitive consumers at price $1 + d$, earning profit $\beta(1 + d)$, or (ii) sell to all consumers at price 1, earning profit $(\beta + 1 - \beta) \cdot 1 = 1$. The first condition $\beta(1 + d) > 1$ implies that option (i) dominates, so the firm sells only to quality-sensitive consumers. When all consumers know the product quality and the high-quality firm adopts the PP technology, the firm can simultaneously charge quality-sensitive consumers $1 + d$ and quality-insensitive consumers 1, earning profit $\beta(1 + d) + (1 - \beta) \cdot 1 - k$. The second condition $k < 1 - \beta$ (i.e., $\beta(1 + d) + (1 - \beta) \cdot 1 - k > \beta(1 + d)$) then implies that the high-quality firm benefits from adopting the PP technology in the absence of quality uncertainty.

makes purchase decisions according to her belief about product quality, which is based on her observed information about the firm's action $\gamma_i(\sigma)$, using the Bayes' rule whenever applicable. Therefore, we only need to consider the quality beliefs of uninformed quality-sensitive consumers. Throughout the remainder of the paper, unless otherwise specified, any reference to "consumers' quality belief" pertains to this group.

The firm's equilibrium action σ maximizes its profit $\pi_j(\sigma)$, anticipating its impact on consumers' beliefs and purchase decisions. We use σ_j to denote type-j firm's *equilibrium* action in a PBE. For convenience, we let $w_j \equiv w(\sigma_j)$, $p_j \equiv p(\sigma_j)$, and $P_j(\theta,\tau) \equiv P(\sigma_j,\theta,\tau)$ be the corresponding outcomes in the PBE.

If $\gamma_i(\sigma_h) \neq \gamma_i(\sigma_l)$, then consumer i's observed information differs depending on the firm's quality, so the consumer's equilibrium belief will be $\phi(\gamma_i(\sigma_h)) = 1$ and $\phi(\gamma_i(\sigma_l)) = 0$. By contrast, if $\gamma_i(\sigma_h) = \gamma_i(\sigma_l)$, then consumer i's observed information is the same regardless of the firm's quality, so the consumer's equilibrium belief will be the prior $\phi(\gamma_i(\sigma_h)) = \phi(\gamma_i(\sigma_l)) = \lambda$.

In our setting, both pooling and separating PBE can arise: the equilibrium is pooling if $\sigma_h = \sigma_l$, and separating otherwise. In typical signaling games, a firm's action is fully observed by all consumers, so all consumers correctly tell a firm's type in a separating equilibrium. In our setting, however, if the firm uses the PP technology ($w = 1$), a consumer i observes only her price $P(\theta_i,\tau_i)$ but not others' personalized prices, that is, the whole function $P(\theta,\tau)$. As a result, even in a separating equilibrium ($\sigma_h \neq \sigma_l$), uninformed consumers may still be unable to tell a firm's type.[8] We introduce the following concepts to precisely describe consumer equilibrium beliefs. We call a separating equilibrium *individually separating* for consumer i if $P_h(\theta_i,\tau_i) \neq P_l(\theta_i,\tau_i)$ and *individually pooling* for consumer i if $P_h(\theta_i,\tau_i) = P_l(\theta_i,\tau_i)$. Note that these concepts are defined for each individual consumer i and are relevant only in separating equilibria with $w_h = w_l = 1$, because in other scenarios consumers can either directly tell a firm's type ($w_h \neq w_l$) or receive uniform prices from both firm types ($w_h = w_l = 0$).[9]

In our setting, there are multiple PBE supported by different off-equilibrium beliefs, many of which are unreasonable. To rule out unreasonable equilibria and pin down a unique equilibrium, we adopt the *strongly undefeated equilibrium* (SUE) refinement, which has been widely used in the economics and marketing literature (Mezzetti and Tsoulouhas 2000, Gill and Sgroi 2012, Miklós-Thal and Zhang 2013, Guo and Jiang 2016, Subramanian and Rao 2016, Wu et al. 2020, Chen et al. 2025).[10] Other common equilibrium refinements, such as the D1 criterion and the intuitive criterion, can fail to select a unique equilibrium outcome in our setting.[11]

[8] For example, consider a PBE with $w_h = w_l = 1$, and there are two uninformed consumers i_1 and i_2 for whom $P_h(\theta_{i_1},U) = P_l(\theta_{i_1},U)$ but $P_h(\theta_{i_2},U) \neq P_l(\theta_{i_2},U)$. Hence, the equilibrium is separating ($\sigma_h \neq \sigma_l$), but consumer i_1 still cannot tell the firm's type because $\gamma_{i_1}(\sigma_h) = \gamma_{i_1}(\sigma_l)$.

[9] As discussed, because it is a dominant strategy for the firm to set the same price for all consumers of the same type, consumers will also rationally believe so.

[10] For more technical discussions of SUE, see Mezzetti and Tsoulouhas (2000) and Appendix A.1. of Wu et al. (2020). Also, see our Online Appendix A for the technical details of the off-equilibrium beliefs in our analysis.

[11] See OA.3 in Online Appendix A for more details about the analysis.

In our setting, the SUE refinement will select the PBE that, among the universe of PBE supported by any possible off-equilibrium belief, yields the highest profit for the high-quality firm. If multiple equilibria satisfy this criterion, then the refinement will further select among these equilibria the one that yields the highest profit for the low-quality firm. As suggested by Miklós-Thal and Zhang (2013), the SUE, which is the "best equilibrium" from the perspective of the high-quality firm, is intuitively appealing because it is the high-quality firm that wants to reveal its type.

Following the definition of SUE, we solve for the SUE by finding the PBE that yields the highest profit for the high-quality firm among all PBE. If multiple PBE satisfy this criterion, we further choose the PBE that yields the highest profit for the low-quality firm. In our setting, this process will select a unique equilibrium. We will refer to the SUE simply as the *equilibrium* in the rest of the paper. We use the superscript "$*$" over a variable to indicate its equilibrium value. Table 1 summarizes the main notations. All analyses and proofs are given in the Online Appendix.

Table 1. Summary of Notations

Symbol	Meaning
j	Product quality type, which can be either high (h) or low (l)
λ	Prior probability that the product has high quality
α	Proportion of informed consumers
β	Proportion of quality-sensitive consumers
τ_i	Consumer i's informedness about product quality: I—informed; U—uninformed
θ_i	Consumer i's valuation for product quality: S—quality-sensitive; N—quality-insensitive
d	Additional value quality-sensitive consumers derive from the high-quality product
k	Fixed cost of adopting the PP technology
σ	Firm's action, which includes both whether to adopt the PP technology and the corresponding personalized or uniform prices
w	Firm's decision on whether to adopt the PP technology: 0—not adopt; 1—adopt
p	Firm's uniform price for all consumers when it does not adopt its PP technology
$P(\theta,\tau)$	Firm's personalized prices when it adopts its PP technology
$\gamma_i(\sigma)$	Consumer i's observed information when the firm plays σ
$\phi(\gamma_i(\sigma))$	Consumer i's posterior belief about the product quality based on $\gamma_i(\sigma)$
$u(\sigma,\theta_i,\tau_i)$	Consumer i's expected utility of purchasing the product
$\pi_j(\sigma)$	Type-j firm's profit when it plays σ
CS	Ex-ante consumer surplus, $CS=\lambda\cdot CS_h+(1-\lambda)\cdot CS_l$
SW	Ex-ante social welfare, $SW=\lambda\cdot SW_h+(1-\lambda)\cdot SW_l$

4 Analysis

We first examine a benchmark without quality uncertainty, in which all consumers know about product quality. Then, we examine the general scenario with quality uncertainty and compare the results with the benchmark to understand how consumers' quality uncertainty affects the firm's personalized pricing decisions, including whether it adopts the PP technology and its corresponding uniform or personalized price levels. Finally, we study the comparative statics of the welfare outcomes accounting for the firm's strategic personalized pricing decisions.

4.1 Benchmark: no consumer quality uncertainty

In this benchmark, all consumers are informed about product quality ($\alpha = 1$), and the firm's decision on PP technology adoption depends on whether the resulting revenue benefit from price discrimination exceeds the fixed adoption cost k. Under the condition of equation (3), the equilibrium outcome is summarized in the following lemma.

Lemma 1 (PP technology adoption without quality uncertainty)**.** *When all consumers are informed about product quality (*$\alpha = 1$*), the high-quality firm will adopt the PP technology, whereas the low-quality firm will not adopt it. The high-quality firm sets the personalized price for a consumer to her willingness to pay* v_{ij} *and fully extracts the consumer surplus.*

When all consumers are informed about product quality, the high-quality firm will adopt the PP technology, whereas the low-quality firm will not adopt it. The latter is because only the high-quality firm has incentives to price discriminate consumers based on their quality sensitivity. However, as we will show later in Propositions 1–3, in the presence of quality uncertainty, the firm's PP strategy significantly deviates from this perfect-information benchmark. The high-quality firm may refrain from adopting the PP technology, whereas the low-quality firm may choose to adopt it in equilibrium. Moreover, the equilibrium personalized price levels may depart from consumers' willingness to pay.

4.2 General case: with consumer quality uncertainty

Next, we analyze the general case where some consumers are uninformed about quality (i.e., $\alpha < 1$). Uninformed quality-sensitive consumers form their belief about quality based on their observed information about the firm's action, $\gamma_i(\sigma)$. Thus, a low-quality firm has an incentive to mimic a high-quality firm's action to let these consumers wrongly believe that the product has high quality. Meanwhile, the high-quality firm has an incentive to choose an action that is unprofitable for the low-quality firm to mimic, which allows these consumers to correctly infer the high quality.

Depending on whether each firm type will adopt the PP technology and whether different firm types will set the same prices, there are six types of potential equilibrium (see Table A1 in the appendix). However, some of them cannot constitute an equilibrium. For example, we can rule out the scenario in which only the low-quality firm adopts the PP technology ($w_h = 0, w_l = 1$). In this case, consumers can correctly infer the firm's quality from the adoption of the PP technology w in equilibrium, and therefore the low-quality firm can profitably deviate to not using the PP technology to save the adoption cost k.

We prove that only three types of outcomes can constitute an equilibrium. The first type is the *no-adoption* equilibrium, in which the firm does not adopt the PP technology regardless of its quality type ($w_h = w_l = 0$), but the high-quality firm sets a higher uniform price than the low-quality firm does, and thus uninformed quality-sensitive consumers can correctly infer quality based on the uniform price, thereby $\phi(\gamma_i(\sigma_h^*)) = 1, \phi(\gamma_i(\sigma_l^*)) = 0$. The second type is the *partial-adoption* equilibrium, in which only the high-quality firm adopts the PP technology ($w_h^* = 1, w_l^* = 0$). In such an equilibrium, uninformed quality-sensitive consumers can correctly infer quality based on w, thereby $\phi(\gamma_i(\sigma_h^*)) = 1, \phi(\gamma_i(\sigma_l^*)) = 0$. The third type is the *universal-adoption* equilibrium, in which the firm, regardless of its quality type, adopts the PP technology ($w_h^* = w_l^* = 1$). In such an equilibrium, the firm charges the same personalized price for uninformed quality-sensitive consumers regardless of its product quality: $P_h^*(S,U) = P_l^*(S,U)$. Therefore, these consumers cannot distinguish the product quality based on the observed information $\gamma_i(\sigma_h^*) = \gamma_i(\sigma_l^*)$, thereby $\phi(\gamma_i(\sigma_h^*)) = \phi(\gamma_i(\sigma_l^*)) = \lambda$. In other words, the equilibrium is *individually pooling* for uninformed quality-sensitive consumers. The detailed equilibrium prices for each type are given in Table 2.

Table 2. Equilibrium PP adoption and prices

Equilibrium types	Equilibrium prices
No adoption	$p_h^* = \min\{1+d, \frac{1}{(1-\alpha)\beta}\}$
$w_h^* = w_l^* = 0$	$p_l^* = 1$
Partial adoption	$P_h^*(S,I) = 1+d, P_h^*(S,U) = \min\{1+\frac{k}{\beta(1-\alpha)}, 1+d\}, P_h^*(N,I) = 1, P_h^*(N,U) = 1$
$w_h^* = 1, w_l^* = 0$	$p_l^* = 1$
Universal adoption	$P_h^*(S,I) = 1+d, P_h^*(N,I) = 1, P_h^*(S,U) = 1+\lambda d, P_h^*(N,U) = 1$
$w_h^* = w_l^* = 1$	$P_l^*(S,I) = 1, P_l^*(N,I) = 1, P_l^*(S,U) = 1+\lambda d, P_l^*(N,U) = 1$

In sharp contrast to the benchmark, with quality uncertainty, the high-quality firm may strategically abstain from adopting the PP technology (corresponding to the no-adoption equilibrium). The abstention stems from the fact that the adoption of PP, as compared to charging a uniform price, may weaken its effectiveness in signaling its high quality. As established in Bagwell and Riordan (1991), when a high-quality firm does not adopt the PP technology, it can signal high quality with a high uniform price. At this high price, quality-sensitive consumers will not buy the product if they believe or know that the product has low quality. A firm with low product quality would find it unprofitable to mimic the high price because doing so would prevent it from selling to informed quality-sensitive consumers. Instead, it would prefer to capture these consumers as well as quality-insensitive consumers with a low uniform price. Consequently, uninformed quality-sensitive consumers rationally infer high quality from the high uniform price. By contrast, when the high-quality firm adopts the PP technology, charging such a high personalized price to uninformed quality-sensitive consumers can no longer convince them of its high quality. This is because a low-quality firm would be strongly incentivized to also adopt the PP technology and mimic the high personalized price for these consumers. The mimicry does not prevent the firm from selling to other consumers with low personalized prices, which are unobservable to uninformed quality-sensitive consumers. Therefore, in markets with consumers' quality uncertainty, the high-quality firm faces an important *tradeoff* regarding the adoption of the PP technology: doing so allows the firm to price discriminate among consumers but may prevent it from effectively signaling its high quality.

In light of this tradeoff, we proceed to elaborate on the firm's decisions regarding its PP strategy, which is twofold: when to adopt the PP technology and how to set the personalized prices for different consumers. We first consider how the firm's strategy is shaped by the prevalence of consumer quality uncertainty in the market. A larger number of informed consumers α indicates that quality uncertainty is less prevalent in the market. Then, we examine how the firm's strategy is influenced by the quality premium of the high-quality product relative to the low-quality one, which is jointly determined by the number of quality-sensitive consumers β and their additional valuation for the premium features of the high-quality product d. We also examine the impact of PP adoption cost k. Lastly, we discuss the managerial implications.

Impact of prevalence of quality uncertainty. Proposition 1 summarizes the firm's PP adoption decisions in equilibrium, moderated by the number of informed consumers α. Figure 2 further delineates the conditions and decisions.

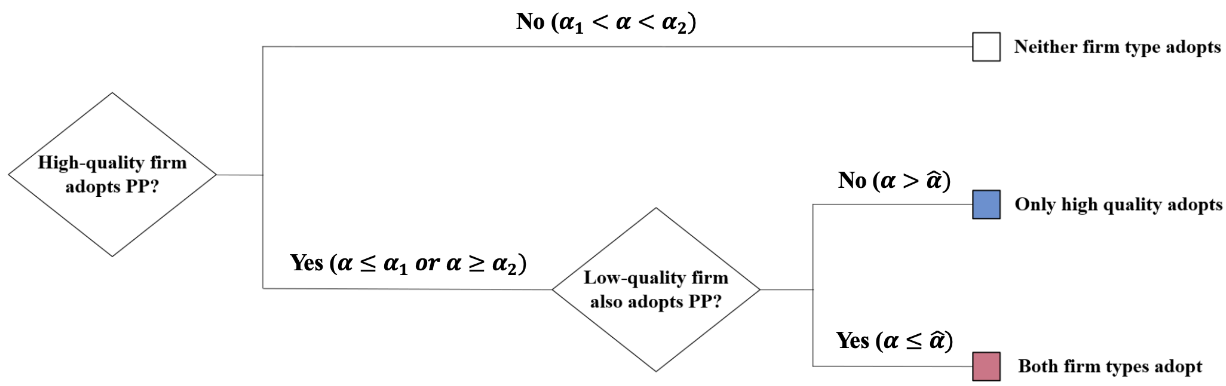
Proposition 1 (PP technology adoption with quality uncertainty)**.** *In equilibrium, the high-quality firm will adopt the PP technology when the number of informed consumers $\alpha \le \alpha_1$ or $\alpha \ge \alpha_2$, with thresholds $\alpha_1 < \alpha_2$; furthermore, in this region, the low-quality firm will also adopt the PP technology when $\alpha \le \hat{\alpha} = 1 - \frac{k}{\beta\lambda d}$.* [12]

Proposition 1 reveals a non-monotonic relationship between the prevalence of consumer quality uncertainty and the high-quality firm's adoption of the PP technology: the high-quality firm refrains from adoption when the number of informed consumers α lies in an intermediate range ($\alpha_1 < \alpha < \alpha_2$), but adopts the PP technology when α is low or high. When α is low, most consumers are uninformed, so the low-quality firm has strong incentives to mimic the high-quality firm's action. In this case, even if the high-quality firm refrains from adoption, it can avoid mimicry only by dramatically reducing its uniform price p_h, leading to a low profit.[13] Consequently, the high-quality firm finds it profitable to adopt the PP technology to price discriminate consumers. By contrast, when α is high, most consumers are informed, so the low-quality firm has weak incentives to mimic the high-quality firm's action. In this case, by adopting the PP technology the

[12] All expressions of the thresholds in the paper are provided in the online appendices.

[13] Mathematically, to avoid mimicry, p_h must guarantee that the low-quality firm's profit of mimicking ($p_h \cdot (1-\alpha)\beta$) is lower than its profit of setting its price to one and selling to all consumers: $p_h \le \frac{1}{\beta(1-\alpha)}$. This price bound decreases as α decreases.

Figure 2. Equilibrium PP technology adoption situations



high-quality firm secures substantial profit from extracting the surplus of most quality-sensitive consumers, who are informed about product quality. Therefore, the high-quality firm also adopts the PP technology to price discriminate consumers. Finally, when α is intermediate, the high-quality firm refrains from adoption to benefit from effective quality signaling.

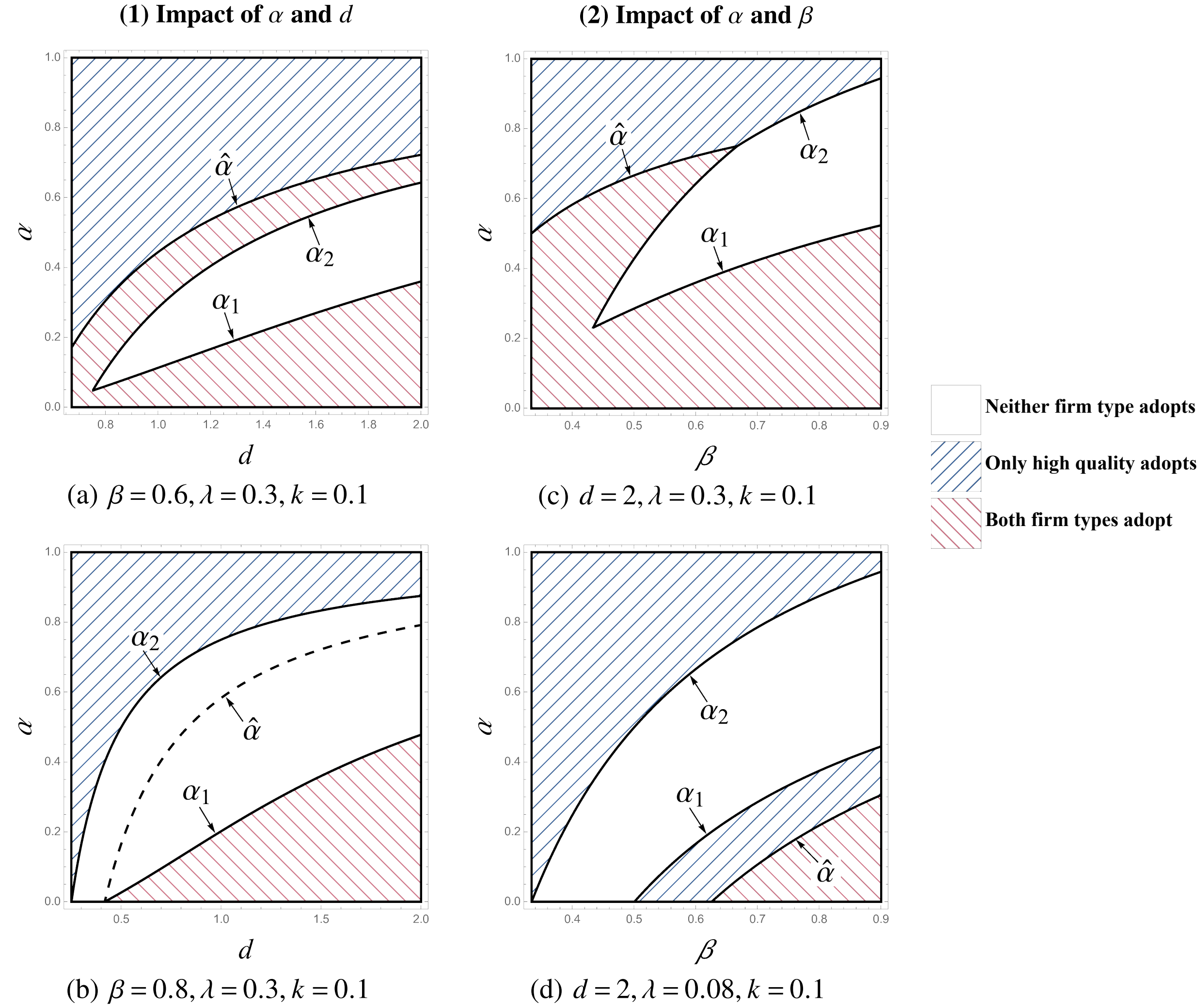
In addition, Proposition 1 shows that in the region where the high-quality firm adopts the PP technology in equilibrium ($\alpha < \alpha_1$ or $\alpha > \alpha_2$), the low-quality firm also adopts it when α is relatively low ($\alpha \leq \hat{\alpha}$), resulting in a universal-adoption equilibrium. Recall that the low-quality firm has incentives to adopt the PP technology only if the high-quality firm also adopts it. Conditional on adoption by the high-quality firm, when α is low, signaling high quality to uninformed quality-sensitive consumers entails a high opportunity cost: the high-quality firm must significantly lower its personalized prices $P_h(S,U)$ to deter mimicry.[14] Therefore, the high-quality firm tolerates the low-quality firm's mimicry.

Figure 3 provides four graphical examples illustrating the equilibrium pattern of PP technology adoption. Consistent with the discussion above, the high-quality firm adopts the PP technology when α is high or low (the blue and red shaded regions); conditional on the high-quality firm's adoption, the low-quality firm also adopts when α is low (the red shaded region).

Next, we elaborate on how a firm that has adopted the PP technology should personalize prices across consumers. Proposition 2 shows that, under quality uncertainty, the high-quality firm may set personalized prices for uninformed quality-sensitive consumers below their WTP, contrary to the standard first-degree price discrimination result under perfect information. Interestingly, quality uncertainty may also allow the low-quality firm to charge uninformed quality-sensitive consumers personalized prices above their WTP for a low-quality product. For all other consumers, the firm charges the full WTP regardless of quality type.

[14] Mathematically, to avoid mimicry, $P_h(S,U)$ must guarantee that the low-quality firm's profit of adopting the PP technology and mimicking the personalized price for uninformed quality-sensitive consumers ($P_h(S,U) \cdot (1-\alpha)\beta + 1 \cdot [1-(1-\alpha)\beta] - k$) is lower than its profit of setting a uniform price of one and selling to all consumers: $P_h(S,U) \leq 1 + \frac{k}{\beta(1-\alpha)}$. This price bound decreases as α decreases.

Figure 3. Equilibrium adoption of the PP technology



Note. The four panels are numerical illustrations of the three equilibrium regimes of PP technology adoption (no, partial, and universal adoption) for the indicated parameter values. Depending on the parameters, not all three regimes need arise.

Proposition 2 (Personalized prices when the firm adopts PP in equilibrium)**.**

(i) *In the parameter region yielding a partial-adoption equilibrium ($w_h^* = 1, w_l^* = 0$), when $\alpha < 1 - \frac{k}{\beta d}$,*[15] *the high-quality firm charges uninformed quality-sensitive consumers a personalized price lower than their WTP for a high-quality product: $P_h^*(S,U) = 1 + \frac{k}{\beta(1-\alpha)} < 1 + d$.*

(ii) *In the parameter region yielding a universal-adoption equilibrium ($w_h^* = w_l^* = 1$), the firm, regardless of its quality type, charges uninformed quality-sensitive consumers the same personalized price, which is between their WTP for a high-quality product and their WTP for a low-quality product: $1 < P_h^*(S,U) = P_l^*(S,U) = 1 + \lambda d < 1 + d$.*

[15] The condition can be equivalently rewritten as $\beta > \frac{k}{d(1-\alpha)}$, $d > \frac{k}{\beta(1-\alpha)}$, or $k < (1-\alpha)\beta d$.

(iii) The firm, when adopting the PP technology, charges other consumers a personalized price equal to their WTP.

First, in a *partial-adoption* equilibrium, even though uninformed quality-sensitive consumers correctly infer product quality from w, the high-quality firm may set personalized prices for these consumers below their WTP for a high-quality product. This occurs when the share of informed consumers α is low.[16] The intuition is that when most consumers are uninformed, the low-quality firm has strong incentives to adopt the PP technology and mimic the high-quality firm's $P_h(S,U)$ to sell to uninformed quality-sensitive consumers. To deter mimicry, the high-quality firm must lower $P_h(S,U)$ sufficiently to make imitation unprofitable in equilibrium. Moreover, as α decreases, the incentive to reduce this price grows stronger.

Second, in a *universal-adoption* equilibrium, uninformed quality-sensitive consumers receive the same personalized price regardless of the firm's quality type; consequently, they cannot infer product quality in equilibrium. In this case, the equilibrium price for these consumers, $1+\lambda d$, equals their expected WTP based on their prior quality belief.

Importantly, Propositions 1 and 2 reveal that in markets with consumers' quality uncertainty, when the high-quality firm adopts the PP technology, it should not simply follow the doctrine of charging consumers their WTP. Instead, the firm may need to calibrate the price levels based on the prevalence of quality uncertainty among consumers to optimally balance the dual incentives of extracting surplus and signaling quality.

Impact of quality premium. As shown in Figure 3, the firm's PP technology adoption is also influenced non-monotonically by the number of quality-sensitive consumers (β) and their additional valuation for the premium features of the high-quality product (d). These two parameters jointly determine the average *quality premium* of the high-quality product over the low-quality one. Proposition 3 summarizes the result.

Proposition 3 (Impact of quality premium on PP technology adoption)**.** *In equilibrium, the high-quality firm will adopt the PP technology when the size of quality-sensitive consumers $\beta \le \beta_1$ or $\beta \ge \beta_2$ (where $\beta_1 < \beta_2$), or equivalently when their additional valuation for the high-quality product's premium features $d \le d_1$ or $d \ge d_2$ (where $d_1 < d_2$); furthermore, in this region, the low-quality firm will also adopt the PP technology in equilibrium when $\beta > \hat{\beta}$ or $d > \hat{d}$.*

Proposition 3 shows that given the number of informed consumers (α), the high-quality firm will refrain from adopting the PP technology if the high-quality product's premium features are moderately favorable ($\beta_1 < \beta < \beta_2$ or $d_1 < d < d_2$). The intuition is similar to the earlier discussion about Proposition 1. When β or d is high, the low-quality firm has strong incentives to mimic the high-quality firm's action. In this situation, even if the high-quality firm did not adopt PP, it would still need to significantly lower its uniform price to avoid mimicry. Therefore, adopting PP still yields a higher equilibrium profit for the high-quality firm.

[16] This condition holds in certain parameter regions, e.g., the blue shaded region of Figure 3 (a) where $\hat{\alpha} = 1 - \frac{k}{\beta\lambda d} < \alpha < 1 - \frac{k}{\beta d}$.

Conversely, when β or d is low, the low-quality firm has weak incentives to mimic the high-quality firm's action, leading to little harm to the high-quality firm's profitability of using personalized pricing. Thus, the high-quality firm still tends to adopt PP. Otherwise, the high-quality firm forgoes price discrimination and benefits from effective quality signaling. Furthermore, conditional on the high-quality firm's adoption, when β or d is sufficiently high, the low-quality firm has strong mimicking incentives and thus will also adopt the PP technology and mimic the high-quality firm's price for uninformed quality-sensitive consumers in equilibrium. Regarding the personalized price levels, as suggested by Proposition 2, in a partial-adoption equilibrium, the high-quality firm may set the personalized price for uninformed quality-sensitive consumers lower than their WTP when β or d is high. Similarly, this is because the high-quality firm needs to downwardly tweak $P_h(S,U)$ from $1+d$ to suppress the low-quality firm's mimicking incentives. Moreover, when there are more quality-sensitive consumers in the market (a larger β), the high-quality firm tends to cut the price level more.

In summary, our analysis highlights that a high-quality firm should refrain from adopting personalized pricing when the number of consumers who know product quality (α) is intermediate, when the number of consumers who appreciate the premium features of the high-quality product (β) is intermediate, and when their appreciation for the high-quality features (d) is moderate.

Finally, we note that the main qualitative message, that the high-quality firm may refrain from adopting PP in equilibrium when β, d, and α are moderate, does not rely on the SUE refinement. Indeed, we find that when β, d, and α are moderate, the no-adoption equilibrium selected by SUE can be the unique equilibrium surviving the D1 criterion, as formalized in Lemma 2.[17]

Lemma 2 (No-adoption equilibrium can uniquely survive D1)**.** *The no-adoption equilibrium, in which* $\sigma_h^* = (w_h^* = 0, p_h^* = \min\{\frac{1}{(1-\alpha)\beta}, 1+d\})$ *and* $\sigma_l^* = (w_l^* = 0, p_l^* = 1)$*, can uniquely survive the D1 criterion when* β*,* d*, and* α *are moderate.* [18]

Impact of PP adoption cost. Additionally, we have examined the impact of PP adoption cost k on the firm's strategy. Intuitively, a larger adoption cost will discourage the firm from adopting the PP technology. In equilibrium, when k is low, both the high-quality firm and the low-quality firm will adopt PP; when k is high, either only the high-quality firm will adopt PP, or neither type of the firm will adopt PP. Also, Proposition 2 implies that in a partial-adoption equilibrium, the high-quality firm may downwardly tweak the personalized price $P_h(S,U)$ from $1+d$ when k is low, so as to weaken the low-quality firm's mimicking incentives. Moreover, when adopting PP becomes less costly (a smaller k), the high-quality firm tends to lower the price level more.

[17] Because the D1 criterion need not select a unique equilibrium for general parameter values, we did not characterize the full parameter thresholds or conduct comparative statics analysis on market outcomes, or compare with such results in the main model.

[18] In Online Appendix A, we show a sufficient condition $\max\left\{\frac{k}{(1-\alpha)d}, \frac{1}{1+(1-\alpha)d}\right\} < \beta < \min\left\{\frac{1}{(1-\alpha)d}, \frac{1}{(1-\alpha)(1+\lambda d)}, \frac{k}{(1-\alpha)\lambda d}\right\}$, which can be equivalently expressed as moderate ranges of α and d.

Managerial implications. Our findings suggest that, when consumers are uncertain about product quality, a high-quality firm should weigh the two roles of personalized pricing, extracting surplus and signaling quality, in deciding whether to adopt it. Specifically, the high-quality firm should abstain from using personalized pricing when an intermediate number of consumers are uninformed about product quality. Such situations might arise, for instance, when the product has been introduced to the market for some time—allowing quality information to spread among some consumers through reviews, word-of-mouth, or advertising—but has not yet achieved broad market penetration. Similarly, when the product requires a moderate level of consumer effort or expertise to access and interpret quality information (e.g., technical product specifications, consumer reviews), quality uncertainty may persist for a substantial fraction of the market. In practice, firms can gauge the informed consumer share based on how readily quality information diffuses in the product category (e.g., whether performance attributes are readily verifiable, the prevalence of expert testing or standardized benchmarks, and historical diffusion patterns for comparable products).

Moreover, our result implies that a high-quality firm can proactively improve the profitability of using personalized pricing by strategically investing in consumer education. For example, the firm can help consumers better recognize its product quality through efforts such as providing clear and easily understandable information on the product page, running advertising campaigns that highlight key quality features, partnering with reputable social media influencers who can credibly endorse the product, and integrating consumer reviews from other trusted sources. These actions can accelerate the market's transition from partial to widespread quality awareness. Doing so can enable the firm to re-establish personalized pricing as a profitable strategy by alleviating the tension between surplus extraction and quality signaling.

In addition, a high-quality firm can also assess the extent of quality improvements offered by premium product features and align its pricing practices with the strength of its quality differentiation. The firm tends to benefit from adopting personalized pricing when the premium features represent either a substantial improvement or only a minor improvement over existing products. Lastly, a firm should also adjust its personalized pricing adoption and price levels to technological improvements that reduce the cost of implementing personalized pricing.

Finally, we emphasize that personalized pricing may also serve other purposes, such as learning about demand heterogeneity and segment composition. Our analysis abstracts from these motives to isolate the signaling role. When such benefits outweigh the adverse signaling effect shown in our paper, high-quality firms may nonetheless find it profitable to adopt personalized pricing.[19]

4.3 Welfare impacts of the PP technology

In this section, we study how various market factors affect the welfare of different market parties under the firm's equilibrium PP strategies.

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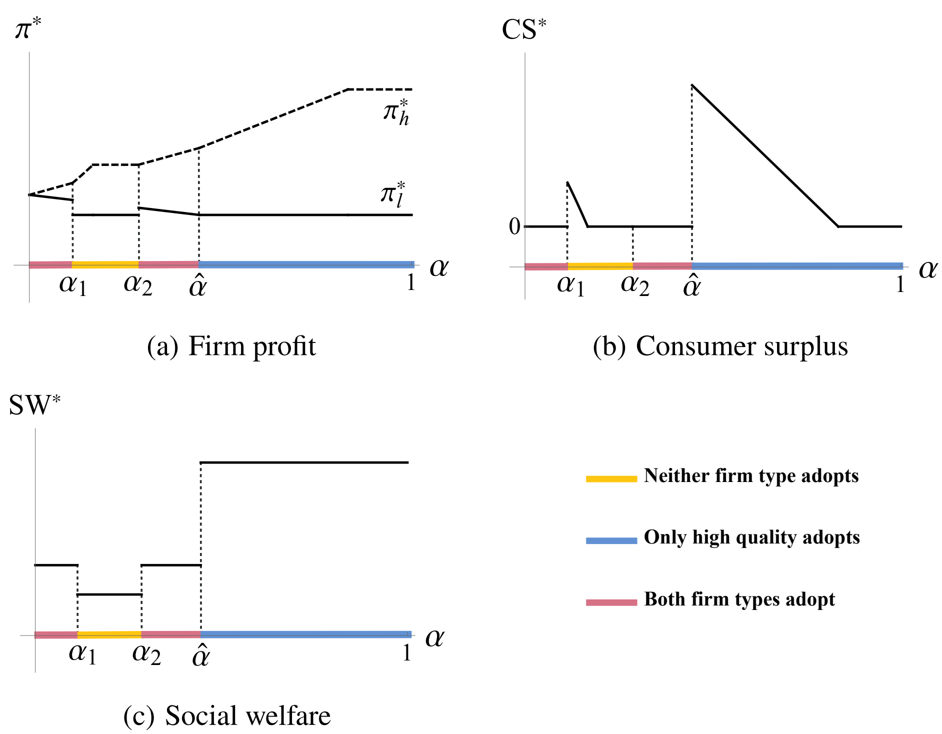
[19] We thank an anonymous reviewer for suggesting this point.

First, in today's markets, product quality information, such as product review blogs or videos from professional content creators or regular consumers, has become more accessible to consumers than ever. The increased information availability is reflected as an increase in the number of informed consumers α in our model. One may intuit that when α increases, consumer surplus will increase since more consumers know about product quality; correspondingly, the high-quality firm should become better off, whereas the low-quality firm should be worse off. Interestingly, Proposition 4 shows that an increase in α can increase the low-quality firm's profit but reduce consumer surplus and social welfare in equilibrium.[20]

Proposition 4 (Welfare impacts of quality uncertainty prevalence)**.** *As the fraction of informed consumers α increases, (i) the high-quality firm's equilibrium profit weakly increases, and the low-quality firm's equilibrium profit can also increase; (ii) equilibrium consumer surplus can decrease; and (iii) equilibrium social welfare can decrease.*

Figure 4. Welfare impacts of quality uncertainty prevalence



(a) Firm profit (b) Consumer surplus

(c) Social welfare

Note. $d = 1$, $\beta = 0.6$, $\lambda = 0.3$, and $k = 0.1$. The figure is a numerical illustration for these parameter values; the equilibrium regimes that arise may differ under other parameter values.

Figure 4 offers an example illustrating the welfare impacts of α. The counterintuitive welfare outcomes arise from changes in the firm's strategic PP adoption decisions when α increases. For example, the low-quality firm's profit increases when α increases from below α_2 to above it, as the equilibrium switches from

[20] We evaluate ex-ante consumer surplus, $CS^* = \lambda CS_h^* + (1-\lambda)CS_l^*$, and ex-ante social welfare, $SW^* = \lambda SW_h^* + (1-\lambda)SW_l^*$, accounting for scenarios where the product quality turns out to be high or low.

a no-adoption one, where the high-quality firm refrains from adoption to signal its quality, to a universal-adoption one, where the high-quality firm adopts the technology to price discriminate consumers and the low-quality firm also adopts it and mimics the high-quality firm's personalized price for uninformed quality-sensitive consumers. As such, the low-quality firm profits more from these consumers while ensuring the same profit from other consumers. In contrast, an increase in α may reduce consumer surplus. For example, when α is slightly higher than $\hat{\alpha}$ in Figure 4, the equilibrium is a partial-adoption one, where the high-quality firm adopts the PP technology and lowers the personalized price for uninformed quality-sensitive consumers from their willingness to pay to avoid the low-quality firm's mimicry: $P_h^*(S,U) = 1 + \frac{k}{\beta(1-\alpha)} < 1 + d$ (see Proposition 2). When α increases, the low-quality firm's mimicking incentives become weaker, so the high-quality firm can lower $P_h^*(S,U)$ to a lesser extent, thereby reducing consumer surplus.

Moreover, social welfare may also decrease with α. For example, when α increases passing α_1, the high-quality firm switches from adopting the PP technology and selling to all consumers, to not adopting it and charging a uniform price $p_h^* > 1$, leaving quality-insensitive consumers unserved.

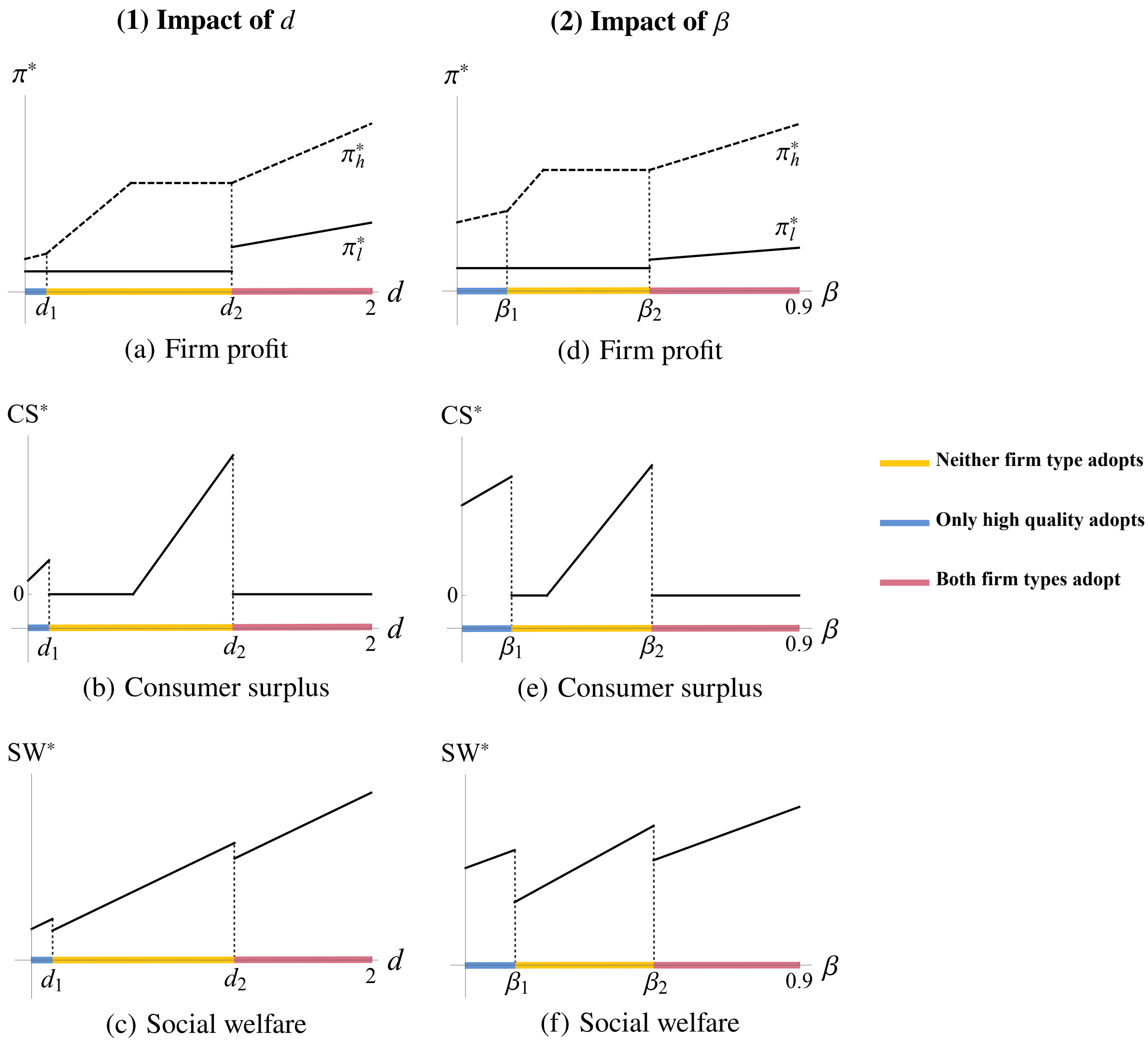
Next, we examine how the quality premium, captured by higher β or d, affects welfare outcomes. A natural conjecture is that a larger quality premium should benefit consumers and raise social welfare, while hurting the low-quality firm. Corollary 1 shows that the opposite can occur. Figure 5 provides a graphical illustration.

Corollary 1 (Welfare impacts of quality premium)**.** *As the number of quality-sensitive consumers increases or when these consumers have higher valuation for the high-quality product (i.e., β or d increases), (i) the high-quality firm's equilibrium profit weakly increases; the low-quality firm's equilibrium profit can also increase; (ii) the equilibrium consumer surplus and social welfare can decrease.*

Consumers can be worse off when the high-quality firm switches from not adopting the PP technology (no-adoption equilibrium) to adopting it (universal-adoption equilibrium) to price discriminate quality-sensitive consumers. This can happen, for example, when β increases passing β_2 or, similarly, when d increases passing d_2. In this case, however, the low-quality firm can be better off because it can pool with the high-quality firm and charge a higher personalized price for uninformed quality-sensitive consumers in equilibrium. Moreover, social welfare can decrease when the high-quality firm switches from adopting the PP technology (partial-adoption equilibrium) to not adopting it (no-adoption equilibrium) and charging a uniform price $p_h^* > 1$, leaving quality-insensitive consumers unserved. This can occur, for example, when β increases passing β_1 or when d increases passing d_1.

Lastly, we also find that when adopting PP becomes more costly, consumer surplus and social welfare can increase. As the PP adoption cost increases, the firm can switch from adopting PP (universal-adoption equilibrium) to not adopting it (no-adoption equilibrium), leaving quality-insensitive consumers unserved. However, in the no-adoption equilibrium, the high-quality firm may charge a relatively low uniform price,

Figure 5. Welfare impacts of quality premium



Note. For column (1), $\alpha = 0.3$, $\beta = 0.8$, $\lambda = 0.3$, and $k = 0.1$. For column (2), $\alpha = 0.3$, $d = 2$, $\lambda = 0.15$, and $k = 0.1$. The figure is a numerical illustration for these parameter values; the equilibrium regimes that arise may differ under other parameter values.

yielding a positive surplus for quality-sensitive consumers. Moreover, the firm saves the PP adoption cost, increasing social welfare. See Figure A1 in the appendix for an example.

In summary, our findings suggest that, once we account for the firm's strategic adoption of personalized pricing, seemingly beneficial changes in market conditions (such as greater accessibility to product quality information, improvements in product quality, and reductions in personalization costs) may harm consumers and society. Policymakers and consumer advocates should therefore be mindful of the firm's strategic pricing and signaling response when promoting innovations that enhance product quality, improve quality information transparency, or lower personalization costs. Our results also imply that a low-quality firm may benefit from greater word-of-mouth activity, which expands the mass of informed consumers. Consequently, such a firm may not always need to invest in costly efforts to suppress or counteract negative word-of-mouth about its quality.

5 Extensions

5.1 PP with imperfect targeting

In our main model, when the firm adopts the PP technology, it observes the full consumer profile, including the willingness to pay θ_i and the quality informedness τ_i, and thus can personalize prices based on both dimensions. This contrasts with the traditional personalized pricing literature, which often assumes that price discrimination is based only on consumers' quality valuation. Additionally, in many practical scenarios, firms can more easily obtain consumers' information about their quality preferences than their quality informedness. This section shows that our main results remain qualitatively robust in this imperfect targeting scenario, where the firm observes a consumer's θ_i but not τ_i. Also interestingly, the high-quality firm's profit may be higher in this imperfect targeting setting than in the main model with perfect targeting.

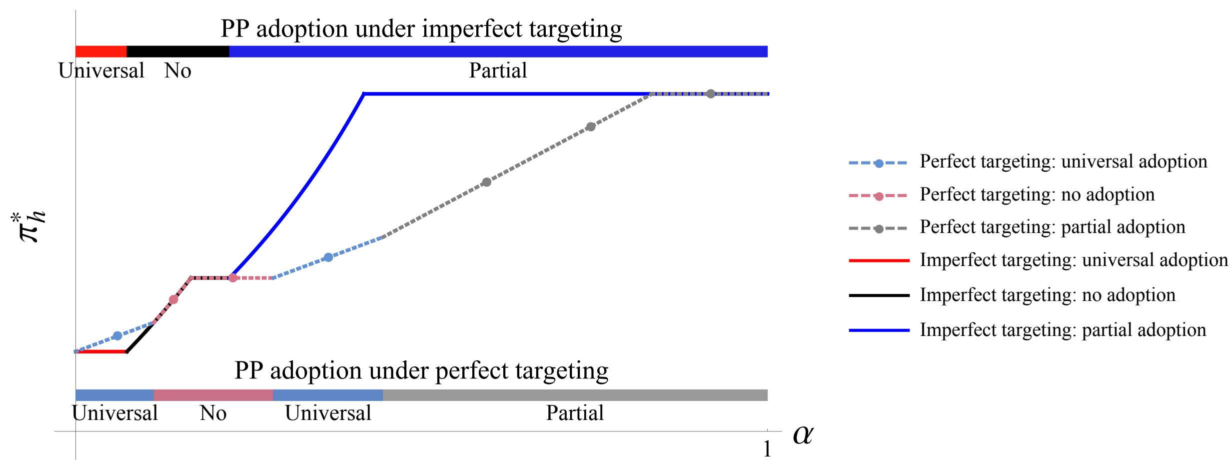
In this extended model, if the firm chooses to adopt the PP technology ($w_j = 1$), given a consumer's quality sensitivity θ_i, the firm's personalized price $P(\theta_i)$ is the same regardless of her quality informedness. Other model setups remain unchanged as in the main model. The detailed analysis is in Online Appendix B.[21] The main findings are qualitatively similar to those in the main model. In particular, the high-quality firm still faces the key tradeoff between adopting the PP technology to price discriminate consumers and refraining from adoption to more efficiently signal its high quality. Moreover, the firm's patterns of PP technology adoption are qualitatively unchanged compared with those in the main model with perfect targeting (Propositions 1 and 3). The high-quality firm will not adopt the PP technology when the number of informed consumers (α) is intermediate, when the size of quality-sensitive consumers (β) is intermediate, and when the additional valuation for the high-quality product's premium features (d) is moderate. Conditional on the high-quality firm's adoption, the low-quality firm will also adopt it when α is low, and when β and d are high. Similar to Proposition 2, when adopting PP, the high-quality firm may still set personalized prices lower than consumers' WTP to signal quality. Additionally, the firm's strategic PP adoption can lead to counterintuitive welfare outcomes analogous to those in the main model. For example, both the ex ante consumer surplus and social welfare may decrease with the number of informed consumers α.

We further compare the market outcomes between the perfect targeting scenario (i.e., the main model) and the imperfect targeting one to understand how the firm's adoption of PP technology depends on what information it knows about consumers. Armstrong (2006) suggests that the PP technology increases the firm's profit more when it is based on more granular consumer information. In sharp contrast, Proposition 5 shows that with consumers' quality uncertainty, the high-quality firm's equilibrium profit can be higher in the imperfect targeting scenario than in the perfect targeting one.

[21] In this extended model, a pure-strategy PBE may not exist. To demonstrate robustness, we restrict attention to parameter values where a pure-strategy PBE exists, for which a sufficient condition is $d \leq 3$.

Proposition 5 (Knowing consumer informedness can reduce firm profit)**.** *When the size of informed consumers $\alpha \in (\bar{\alpha}_1, \bar{\alpha}_2)$, the high-quality firm's equilibrium profit, as well as the firm's ex ante equilibrium profit, is lower in the perfect targeting scenario than in the imperfect targeting scenario.*

Figure 6. Comparison of the high-quality firm's equilibrium profit



Note. $d = 1$, $\beta = 0.6$, $\lambda = 0.3$, and $k = 0.1$. The figure is a numerical illustration for these parameter values; the equilibrium regimes that arise may differ under other parameter values.

This counterintuitive result arises from the fact that in the imperfect targeting scenario, if the firm adopts the PP technology, the personalized price for quality-sensitive consumers is independent of their informedness, in contrast to the case where their personalized prices are contingent on informedness in the perfect targeting scenario. This affects the high-quality firm's incentives for PP adoption in two opposite ways. On the one hand, with imperfect targeting, the high-quality firm, to avoid the low-quality firm's mimicry, needs to lower its personalized price for all quality-sensitive consumers, reducing its profit from not only uninformed but also informed quality-sensitive consumers. In comparison, in the perfect targeting scenario, the high-quality firm only needs to lower the personalized price for uninformed quality-sensitive consumers, without reducing its profit from informed quality-sensitive consumers. This effect makes the PP adoption *less beneficial* to the high-quality firm under imperfect targeting than under perfect targeting. On the other hand, with imperfect targeting, the low-quality firm has weaker incentives to mimic the high-quality firm's personalized price for quality-sensitive consumers, which satisfies $P(S) > 1$;[22] such mimicry would lose the demand from informed quality-sensitive consumers, who would buy the product if the low-quality firm instead set a uniform price of one. Therefore, the high-quality firm can lower $P(S)$ to a lesser extent while still avoiding mimicry. This effect makes the PP adoption *more beneficial* to the high-quality firm

[22] The high-quality firm will not set $P(S) \leq 1$, because it can profitably deviate to charging a uniform price of one.

under imperfect targeting than under perfect targeting. Proposition 5 shows that the latter effect dominates the former when the number of informed consumers α is intermediate. See Figure 6 for a graphical example.

Proposition 5 also shows that the firm's ex ante equilibrium profit can be higher in the imperfect targeting scenario than in the perfect targeting scenario. This implies that a firm may benefit from deliberately adopting a coarser pricing system before learning whether its product is high or low quality (e.g., when designing its pricing infrastructure prior to product launch). Pre-committing to a less granular pricing system may not only reduce implementation costs, but also relieve future signaling pressures.[23]

Corollary 2 (Adoption of imperfect-targeting PP technology)**.** *There exist parameter regions in which the high-quality firm does not adopt a PP technology with perfect targeting in equilibrium, yet adopts a PP technology with imperfect targeting.*

The above comparison reveals that the firm's profitability of adopting personalized pricing may be impaired when the firm possesses more information about consumers. As an extension of this insight, Corollary 2 shows that the high-quality firm's decision on whether to adopt personalized pricing can also be switched by the specification of the PP technology. Specifically, we find that when the equilibrium is a no-adoption one in the perfect targeting scenario, the equilibrium may be a partial-adoption one in the imperfect targeting scenario. Figure 6 provides an example, with the dashed red line (Perfect targeting: no adoption) versus the solid blue line (Imperfect targeting: partial adoption). Moreover, the high-quality firm's equilibrium profit can be higher in the latter case. Therefore, instead of completely resisting personalized pricing, the high-quality firm could opt for a constrained PP technology. This result echoes Wang et al. (2023) in that an imperfect personalized pricing algorithm can outperform a perfect one, yet through different channels. In Wang et al. (2023), the effect is driven by consumer search: consumers must incur search costs to make a purchase, and algorithmic errors can give high-valuation consumers a chance to be misclassified as low-valuation consumers and obtain a low price, thereby encouraging more consumers to search and purchase. By contrast, in our setting, the reason is that an imperfect pricing algorithm inhibits the firm from personalizing prices contingent on consumer informedness, thereby reducing the low-quality firm's incentives to mimic the high-quality firm.

In summary, we show that our results are qualitatively robust when the firm does not observe consumers' knowledge about product quality. Moreover, using such information to set personalized prices may reduce a high-quality firm's profit when some consumers are uncertain about product quality. Accordingly, even when richer consumer data or more advanced algorithms make it feasible to identify which consumers are informed about quality, a high-quality firm may prefer not to condition its prices on this information.

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[23] We thank an anonymous reviewer for suggesting this point.

5.2 Using list prices together with personalized prices

In practice, firms adopting personalized prices sometimes also use a list price, which effectively serves as a self-imposed universal price cap for all consumers (Xu and Dukes 2022). For example, in many personalized discount programs, products are available for purchase at their list prices, and consumers receive personalized discounts off the list prices. As we have illustrated in the main analysis, adopting the PP technology may impede a firm's ability to signal its high quality because consumers cannot observe the personalized prices for other consumers. One may wonder whether a firm can facilitate the signaling of its high quality by accompanying its personalized prices with a list price, which lets consumers know the price cap for other consumers.

This extension explores this possibility. In this extended model, if the firm adopts the PP technology ($w = 1$), it can set a list price $\bar{p}$ besides deciding the personalized prices $P(\theta, \tau)$. The list price $\bar{p}$ is observed by all consumers and is a price cap for personalized prices: $P(\theta_i, \tau_i) \leq \bar{p}$ for all i. (We do not consider list prices if the firm does not adopt the PP technology, in which case list prices are a moot concept.) We constrain $\bar{p} \in (1, 1+d) \cup \{\varnothing\}$, where $\bar{p} = \varnothing$ means that the firm does not use a list price (or sets a non-binding list price).[24] The reason is twofold: first, consumers' highest willingness to pay is $1 + d$, so any $\bar{p} \geq 1 + d$ is non-binding; second, if $\bar{p} \leq 1$, then the firm would profitably deviate to not adopting the PP technology and set a uniform price of one, under which all consumers will buy. Other model setups are the same as the main model in Section 4. The detailed analysis is given in Online Appendix C.

Proposition 6 (List prices are not used in equilibrium)**.** *The firm will not use a list price in equilibrium. The equilibrium outcomes are identical to when the firm cannot use list prices (Table 2).*

Proposition 6 suggests that in our context, the firm will not use a list price together with personalized prices in equilibrium, showing the robustness of our main findings. The reason is as follows. The firm will use a list price only under a universal-adoption equilibrium ($w_h^* = w_l^* = 1$); otherwise, when only one firm type adopts the PP technology, consumers can already tell the firm's type from w, so the firm has no incentives to impose a price cap on its personalized prices. We show that in this universal-adoption situation, the high-quality firm cannot use a list price to improve its profit without being mimicked by the low-quality firm. Intuitively, this is because compared with the low-quality firm, the high-quality firm tends to charge higher personalized prices, but the use of a list price implies that the personalized prices are low.

The finding is in sharp contrast to Xu and Dukes (2022), who find that when a firm uses personalized prices, it can signal its private information to consumers by also using a list price. In their setting, the firm's private information (the firm's type) is about whether consumers underestimate or overestimate their valuations for the firm's product. The firm can use a list price in addition to personalized prices to signal

[24] The tie-breaking rule is that if the firm is indifferent between choosing $\bar{p} < 1 + d$ and $\bar{p} = \varnothing$, it will choose the latter (i.e., not using the list price). The tie-breaking rule does not affect the qualitative outcome.

to consumers that they underestimate their valuations, because the firm tends to set *low* personalized prices in the case of underestimation. In other words, in their setting, the type of firm that wants to reveal its type tends to set *lower* personalized prices. By contrast, in our setting, the firm's private information is about its product quality, so the type of firm that wants to reveal its type tends to set *higher* personalized prices. Therefore, if the high-quality firm uses the PP technology, using a list price does not help the firm signal its high quality. We note that one should interpret Proposition 6 as that using list prices cannot help a firm signal its high quality when it adopts the PP technology, instead of that it is never profitable for firms to use list prices in any context. For example, using list prices can be profitable in the context of Xu and Dukes, and firms may use list prices for other reasons, such as signaling the targeting cost in offering personalized prices (Yu and Zhang 2025).

5.3 Using advertising as a quality signaling tool

In practice, firms may use a variety of instruments to convey quality, such as costly advertising, warranties, and other brand or service commitments. However, these instruments are not always salient or readily deployable, and consumers' quality beliefs are often shaped by prices, which are often directly observable during the shopping experience. In many online shopping contexts, consumers first encounter a product while browsing and evaluating, and may not have seen or recalled any advertising for it at that point (e.g., advertising run on other channels that consumers may not have seen). Warranties can similarly serve a signaling role, but are often set at the brand level or governed by standardized policies, making them difficult to adjust at the product level. While online reviews are important sources of quality information, they can be subject to inflation or manipulation in certain product categories, reducing their informational value. In such settings, where non-price signals may be unavailable, hard to attribute, or too coarse, prices become a natural and immediate channel through which consumers form quality beliefs.

That said, our paper is not intended to suggest that quality signaling through PP adoption/non-adoption "replaces" other signaling instruments. Rather, our analysis identifies a distinct channel through which personalized pricing reshapes consumers' quality beliefs, and this channel can remain relevant even when additional signals are available. In Online Appendix G, we formally consider an extended model in which the firm can also choose to incur a fixed cost $f > 0$ to advertise, which can have a potential impact on uninformed consumers' perceptions of quality (e.g., Milgrom and Roberts 1986, Guo and Wu 2016). We show that in equilibrium, even when the high-quality firm uses advertising, the firm may still refrain from adopting the PP technology, because doing so can help the firm more efficiently signal its high quality. This result shows the robustness of the signaling channel we highlight in this paper.

6 Experimental Evidence

We conducted four randomized online experiments to test the premise that personalized pricing weakens consumers' tendency to infer product quality from price.[25] In each experiment, 200 participants imagined shopping for a product whose quality is uncertain. They were told (i) the average price for typical products in the same category, (ii) the focal product's price, and (iii) the focal product's pricing format. Across all experiments, we held the numeric price points constant and randomly assigned participants to one of two pricing formats: uniform price vs. personalized price. Participants rated the focal product's quality relative to the market average on a 7-point scale (1 = "much lower quality than average," 4 = "about average," 7 = "much higher quality than average"). To demonstrate robustness, we varied both the product category and the relative position of the focal price to the market average.

In three studies, the focal product was priced above the market average (wireless earbuds: \$150 > \$100; running shoes: \$200 > \$100; robot vacuum: \$700 > \$400). Consistently, our experimental results showed that participants in the uniform pricing group perceived significantly higher quality than those in the personalized pricing group. That is, a uniform high price was treated as a stronger signal of above-average quality than the same high price labeled as personalized. In a fourth study, the focal product was priced below the market average (wireless earbuds: \$50 < \$100). Here, the pattern reversed: participants in the uniform pricing group perceived significantly lower quality than those in the personalized pricing group.[26] Taken together, these experiments provide convergent evidence for the premise underlying our model: a personalized price is a weaker quality signal than a uniform price, which leads to our key insight that a high-quality firm may strategically refrain from adopting personalized pricing under quality uncertainty.

To complement the statistical results, we present illustrative excerpts from participants' open-ended responses. Several participants in the personalized pricing group explicitly described how personalization weakened their quality judgments. For example, when the focal product was priced above the market average, some participants were less inclined to infer high quality from the high personalized price:

- "I am thinking that the higher price would likely mean higher quality but my confidence is decreased by the personalized price."
- "They are probably charging more because of the personalization aspect. I doubt they are any better than the average priced pair."
- "Since the pricing is personalized and not everyone may be quoted the same price for the same shoes, it makes me skeptical that the shoes are that much higher in quality."

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[25] See Online Appendix I for the details of experiment designs and results.

[26] For each study, we report the mean quality rating (M, on a 7-point scale) and the number of participants (N) for the personalized and uniform pricing conditions, denoted by subscripts PP and UP respectively, as well as the 95% confidence interval (CI) and the p-value for the mean difference (personalized minus uniform) from a two-sided t-test with unequal variances (Welch's t-test): (1) $M_{PP} = 4.73$, $N_{PP} = 85$, $M_{UP} = 5.21$, $N_{UP} = 81$, CI $[-0.758, -0.203]$, $p = .001$; (2) $M_{PP} = 4.74$, $N_{PP} = 93$, $M_{UP} = 5.42$, $N_{UP} = 83$, CI $[-0.992, -0.367]$, $p < .001$; (3) $M_{PP} = 4.96$, $N_{PP} = 89$, $M_{UP} = 5.37$, $N_{UP} = 89$, CI $[-0.738, -0.094]$, $p = .012$; and (4) $M_{PP} = 3.05$, $N_{PP} = 86$, $M_{UP} = 2.62$, $N_{UP} = 87$, CI $[0.161, 0.690]$, $p = .002$.

Similarly, when the focal product was priced below the market average, some participants were less inclined to infer low quality from the low personalized price:

- "I assessed the price of a comparable product, considering the fact that I may have received a discount due to personalized pricing."
- "I feel like it should be of comparable quality. Just because it's a deal doesn't mean it's not a good product."

These responses illustrate the premise underlying our model: personalized pricing can undermine a firm's ability to signal high quality through price.

7 Conclusion

The rapid development of marketing analytics and artificial intelligence provides firms with unprecedented capabilities to personalize prices for individual consumers. This paper examines a firm's personalized pricing strategy in markets where consumers are uncertain about product quality. In such markets, pricing serves dual roles: extracting surplus and signaling quality. We find that implementing personalized pricing, despite enabling price discrimination, may not efficiently convey quality information to consumers. Therefore, in markets with consumer quality uncertainty, a high-quality firm deciding whether to adopt personalized pricing should weigh the resulting loss in quality signaling against the gain from price discrimination. We also find that firm profit may be higher when the firm does not observe consumers' quality informedness than when it does. This suggests that the firm should be careful about conditioning its prices on consumer informedness, for instance, enabled by collecting more consumer information or deploying more advanced artificial intelligence algorithms.

Our findings suggest that a high-quality firm should refrain from adopting personalized pricing when the number of consumers who know product quality is intermediate, when the number of consumers who appreciate the product's premium features is intermediate, and when their appreciation for those features is moderate. Moreover, when adopting personalized pricing, the high-quality firm should not simply follow the convention of setting prices equal to consumers' willingness to pay for full surplus extraction; instead, it may need to shade prices downward to facilitate quality signaling. After accounting for the firm's strategic pricing decisions, improving product quality, increasing consumer access to quality information, or reducing the cost of personalized pricing can hurt consumers and society.

Our paper identifies several directions for future research. First, we focus on personalized pricing. Increasingly, other marketing mix elements, such as advertising and product design, are also highly personalized. Such personalization similarly prevents consumers from observing which marketing mix other consumers receive, analogously to the personalized pricing setting. For example, with highly targeted advertising, a consumer does not observe the firm's advertising intensity directed at other consumers. Future research could examine how personalization affects the quality signaling roles of different marketing instruments. Second, our paper provides several testable predictions. Future empirical research could validate these predictions using market data.

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Online Appendix

Personalized Pricing with Consumers' Quality Uncertainty

Online Appendix A: Proofs of results for the main model

OA.1. Equilibrium derivation for the main model

One can readily show Lemma 1 for the benchmark case where all consumers know the product quality (i.e., $\alpha = 1$). Recall that the adoption cost of PP technology $k < 1 - \beta$. If the high-quality firm adopts the PP technology, its equilibrium profit is $\beta(1+d) + 1 - \beta - k$. Without adopting the PP technology, the high-quality firm's equilibrium profit is $\beta(1+d)$. Thus, the high-quality firm will adopt the PP technology in the absence of quality uncertainty. By contrast, the low-quality firm will not adopt the PP technology to save the adoption cost k.

We next derive the equilibrium for the main model, where some consumers are uninformed (i.e., $\alpha < 1$). Note that the derivation will directly imply Proposition 1, Proposition 2, and Proposition 3 in Section 4.

Table A1. Potential Equilibrium Cases

	Separating ($\sigma_h \neq \sigma_l$)	Pooling ($\sigma_h = \sigma_l$)
$w_h = w_l = 0$	Case 1: PBE exists, denoted as $sep1$-type	Case 2: PBE exists, denoted as $pool2$-type
$w_h = 1, w_l = 0$	Case 3: PBE exists, denoted as $sep3$-type	N/A
$w_h = w_l = 1$	Case 4: PBE exists, denoted as $sep4$-type	Case 5: No PBE exists
$w_h = 0, w_l = 1$	Case 6: No PBE exists	N/A

Table A1 summarizes the potential equilibrium cases. The equilibrium is separating (pooling) if $\sigma_h \neq \sigma_l$ ($\sigma_h = \sigma_l$). We can quickly rule out two cases. First, there will not exist pooling equilibria with $w_h = w_l = 1$. This is because the two firm types will set different equilibrium personalized prices for informed quality-sensitive consumers, who know the product quality: $P_h(S, I) = 1 + d \neq P_l(S, I) = 1$. Therefore, no PBE exists for Case 5. Second, no PBE exists for Case 6. This is because if the low-quality firm chose $w_l = 1$ in an equilibrium with $w_h = 0$, all consumers could tell its type and its equilibrium profit would hence be $1 - k$. However, it can profitably deviate to charging a uniform price of one, which yields a higher profit of 1. In the rest of the proof, we will examine Cases 1 (separating), 2 (pooling), 3 (separating), and 4 (separating), respectively. We denote the PBE in the four cases as $sep1$-type, $pool2$-type, $sep3$-type, and $sep4$-type, respectively.

Following the definition of SUE, we solve for the equilibrium by finding the PBE that yields the highest profit for the high-quality firm among all PBEs. If multiple PBEs satisfy this criterion, we further choose the PBE that yields the highest profit for the low-quality firm. To do so, we need to define an uninformed consumer's belief when her observed information is off equilibrium, i.e., when $\gamma_i(\sigma) \notin \{\gamma_i(\sigma_h), \gamma_i(\sigma_l)\}$.[1]

[1] This is similar to the off-equilibrium belief in a typical signaling game, but technically different. Unlike typical signaling games, even if the firm plays an off-equilibrium action $\sigma' \notin \{\sigma_h, \sigma_l\}$ in our game, an uninformed consumer can still use Bayes' rule to update her belief as long as her observed information remains unchanged, i.e., $\gamma_i(\sigma') = \gamma_i(\sigma_h)$ or $\gamma_i(\sigma') = \gamma_i(\sigma_l)$.

Because finding the SUE requires us to compare the outcomes of all possible PBEs, we focus on the pessimistic beliefs:

$$\phi(\gamma_i(\sigma)) = 0 \text{ if } \gamma_i(\sigma) \notin \{\gamma_i(\sigma_h), \gamma_i(\sigma_l)\}.$$

This is without loss of generality. The reason is that it allows us to identify all possible PBE outcomes, because if a PBE outcome can be supported by another off-equilibrium belief, then it can also be supported by our proposed off-equilibrium belief. The intuition is that by definition, the SUE refinement selects among all possible PBEs. Pessimistic beliefs allow us to find the full set of PBE outcomes because they generate the weakest incentives for firms to deviate.

The remainder of the proof proceeds in five steps: In Steps (i)-(iv), we respectively examine Cases 1-4; for each case, we find the PBE that yields the highest profit for the high-quality firm; we call such PBE the *"high-type-best" equilibrium* for the corresponding case; we denote them as $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$, $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$, $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$, and $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$, respectively. In Step (v), among these four high-type-best PBE outcomes, we find the equilibrium that yields the highest profit for the high-quality firm, which is the SUE.

To proceed, it is without loss of generality to assume that the firm's prices lie on the range of $[1, 1+d]$ throughout the analysis. The reason is that since consumers' willingness to pay lies between 1 and $1+d$, setting (personalized or uniform) prices strictly below 1 or strictly above $1+d$ is strictly sub-optimal.

Finding the high-type-best PBE for each case

Step (i): Find the high-type-best equilibrium for Case 1 $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$

Let $(\sigma_h^{sep1}, \sigma_l^{sep1}, \phi)$ be an $sep1$-type PBE in Case 1, where $w_h^{sep1} = w_l^{sep1} = 0$ and $\sigma_h^{sep1} \neq \sigma_l^{sep1}$. We find $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ in two steps. In step one, we derive the IC (incentive-compatible) constraints for both firm types, which must be satisfied in equilibrium. In step two, we find $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$—the sep1-type PBE that yields the highest profit for the high-quality firm and satisfies the IC constraints.

Step 1. We start with the low-quality firm. In an $sep1$-type equilibrium, it must hold that $w_l^{sep1} = 0$ and $p_l^{sep1} = 1$, which leads to $\pi_l^{sep1} = 1$. It follows that $p_h^{sep1} \in (1, 1+d]$ must be true in the separating equilibrium.

We derive the low-quality firm's IC constraint. We check all possible deviations by discussing the following three cases. First, the low-quality firm will not deviate to a σ' with $w(\sigma') = 1$, because the best deviation profit is $\pi_l(\sigma') = 1 - k$, which is lower than $\pi_l^{sep1} = 1$. Second, the low-quality firm will not deviate to a σ' with $w(\sigma') = 0$ and $p(\sigma') \neq p_h^{sep1}$. This is because all uninformed consumers will believe that the firm has low quality with probability one and hence its deviation profit must be lower than $\pi_l^{sep1} = 1$. Third, we check the low-quality firm's deviation to $\sigma' = \sigma_h^{sep1}$, i.e., $w(\sigma') = 0$ and $p(\sigma') = p_h^{sep1}$. Under this deviation, all uninformed consumers will believe that the firm has high quality with probability one. Therefore, the low-quality firm's deviation profit will be $\pi_l(\sigma') = (1-\alpha)\beta p_h^{sep1}$. That is, the low-quality firm will only sell to *uninformed quality-sensitive* consumers by mimicking the high-quality firm's action σ_h^{sep1}. In this case, the low-quality firm will not deviate if and only if $(1-\alpha)\beta p_h^{sep1} \le 1$. Summarizing the three cases, the low-quality firm's IC constraint is given by

$$(IC_l^{sep1}) : (1-\alpha)\beta p_h^{sep1} \le 1.$$

Next, we consider the high-quality firm. In equilibrium, all consumers believe that the product has high quality with probability one. Thus, the high-quality firm's equilibrium profit is $\pi_h^{sep1} = \beta p_h^{sep1}$, where $p_h^{sep1} \in (1, 1+d]$. We check all possible deviations by discussing the following cases.

If $p_h^{sep1} = 1+d$, the high-quality firm will not deviate to a σ' with $w(\sigma') = 0$, because the deviation profit must be lower than $\pi_h^{sep1} = \beta(1+d)$. However, if $p_h^{sep1} \in (1, 1+d)$, the high-quality firm may deviate to a σ' with $w(\sigma') = 0$. Specifically, we need to consider a possible deviation σ' with $w(\sigma') = 0$ and $p(\sigma') = 1$, where the deviation profit is $\pi_h(\sigma') = 1$. Moreover, we need to check a deviation σ' with $w(\sigma') = 0$ and $p(\sigma') = 1+d$, which is the best deviation among those deviations with $w(\sigma') = 0$ and $p(\sigma') > p_h^{sep1}$. The deviation profit is $\pi_h(\sigma') = \beta\alpha(1+d)$. Note that the high-quality firm will not deviate to a σ' with $w(\sigma') = 0$ and $p(\sigma') \in (1, p_h^{sep1})$, because the deviation profit $\pi_h(\sigma') = \beta\alpha p(\sigma')$ must be lower than $\pi_h^{sep1} = \beta p_h^{sep1}$.

In addition, we consider a deviation σ' with $w(\sigma') = 1$. Since $\phi(\gamma(\sigma')) = 0$ holds for uninformed quality-sensitive consumers, the best deviation with $w(\sigma') = 1$ should satisfy $P(\sigma', S, I) = 1+d$, $P(\sigma', N, I) = 1$, and $P(\sigma', S, U) = P(\sigma', N, U) = 1$, leading to a deviation profit of $\pi_h(\sigma') = \beta\alpha(1+d) + (1-\beta+\beta(1-\alpha)) - k$.

Note that equation (3) suggests $(1-\beta+\beta(1-\alpha)) > k$. Summarizing the above cases, the high-quality firm's IC constraint is given by

$$(IC_h^{sep1}) : \max\{1, \beta\alpha(1+d) + (1-\beta+\beta(1-\alpha)) - k\} \le \beta p_h^{sep1}.$$

Step 2. We find the high-type-best PBE $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ by accounting for both (IC_l^{sep1}) and (IC_h^{sep1}). We divide the discussion into four parameter regions. Recall that in Case 1, $p_h^{sep1} \in (1, 1+d]$ must be true.

In region 1, $\beta \le \frac{1}{(1-\alpha)(1+d)}$ and $k < 1+\beta\alpha d - \beta(1+d)$. In this region, (IC_h^{sep1}) cannot be satisfied. Thus, no $sep1$-type PBE exists.

In region 2, $\beta \le \frac{1}{(1-\alpha)(1+d)}$ and $k \ge 1+\beta\alpha d - \beta(1+d)$. In this region, one can show that the high-type-best $sep1$-type PBE satisfies $\sigma_h^{sep1*} = (w_h^{sep1*} = 0, p_h^{sep1*} = 1+d)$ and $\sigma_l^{sep1*} = (w_l^{sep1*} = 0, p_l^{sep1*} = 1)$. First, (IC_l^{sep1}) is satisfied $\forall p_h^{sep1} \in (1, 1+d]$. Second, among all the p_h^{sep1} that satisfies (IC_h^{sep1}), $p_h^{sep1} = 1+d$ yields the highest profit for the high-quality firm.

In region 3, $\beta > \frac{1}{(1-\alpha)(1+d)}$ and $k < 1+\beta\alpha d - \frac{1}{1-\alpha}$. In this region, $\frac{1}{(1-\alpha)\beta} < 1+d$, and (IC_l^{sep1}) is satisfied for $p_h^{sep1} \in (1, \frac{1}{(1-\alpha)\beta}]$. However, for $p_h^{sep1} \in (1, \frac{1}{(1-\alpha)\beta}]$, (IC_h^{sep1}) is violated. Thus, no $sep1$-type PBE exists.

In region 4, $\beta > \frac{1}{(1-\alpha)(1+d)}$ and $k \ge 1+\beta\alpha d - \frac{1}{1-\alpha}$. In this region, we show that the high-type-best $sep1$-type PBE satisfies $\sigma_h^{sep1*} = (w_h^{sep1*} = 0, p_h^{sep1*} = \frac{1}{(1-\alpha)\beta})$ and $\sigma_l^{sep1*} = (w_l^{sep1*} = 0, p_l^{sep1*} = 1)$. This is because among all possible p_h^{sep1}, $p_h^{sep1}=\frac{1}{(1-\alpha)\beta}$ yields the highest profit for the high-quality firm; moreover, both (IC_l^{sep1}) and (IC_h^{sep1}) are satisfied under this PBE.

To sum up, the $sep1$-type PBE exists when $k\geq 1+\beta\alpha d-\beta\min\{\frac{1}{(1-\alpha)\beta},1+d\}$. Furthermore, the high-type-best $sep1$-type equilibrium satisfies $\sigma_h^{sep1*}=(w_h^{sep1*}=0,p_h^{sep1*}=\min\{\frac{1}{(1-\alpha)\beta},1+d\})$ and $\sigma_l^{sep1*}=(w_l^{sep1*}=0,p_l^{sep1*}=1)$. In this high-type-best equilibrium, $\pi_h^{sep1*}=\beta\min\{\frac{1}{(1-\alpha)\beta},1+d\}$ and $\pi_l^{sep1*}=1$.

Step (ii): Find the high-type-best equilibrium for Case 2 $(\sigma_h^{pool2*},\sigma_l^{pool2*})$

Let $(\sigma_h^{pool2},\sigma_l^{pool2},\phi)$ be a $pool2$-type PBE in Case 2, where $w_h^{pool2}=w_l^{pool2}=0$ and $\sigma_h^{pool2}=\sigma_l^{pool2}$, hence $p_h^{pool2}=p_l^{pool2}$. We use p^{pool2} to denote the identical pooling price. Below, we focus on the case where $p^{pool2}>1$ and find $(\sigma_h^{pool2*},\sigma_l^{pool2*})$ in two steps.[2] In step one, we derive the IC constraints for both firm types, which must be satisfied in equilibrium. In step two, we find $(\sigma_h^{pool2*},\sigma_l^{pool2*})$—the $pool2$-type PBE that yields the highest profit for the high-quality firm and satisfies the IC constraints.

Step 1. In a $pool2$-type equilibrium, uninformed consumers' quality belief is $\phi(\gamma(\sigma))=\lambda$ if $\sigma=\sigma_j^{pool2}$, and $\phi(\gamma(\sigma))=0$ if $\sigma\neq\sigma_j^{pool2}$. Therefore, in equilibrium, $p^{pool2}\leq 1+\lambda d$, because otherwise the low-quality firm's profit would be zero and the pooling equilibrium would not exist.

We first derive the low-quality firm's IC constraint. Since $p^{pool2}>1$, the low-quality firm's equilibrium profit is $\pi_l^{pool2}=(1-\alpha)\beta p^{pool2}$ by selling to only *uninformed quality-sensitive* consumers. One can readily show that among all possible deviations for the low-quality firm, the best deviation is to charge a uniform price of one and receive a profit of $\pi_l(\sigma')=1$. Thus, the low-quality firm's IC constraint is given by

$$(IC_l^{pool2}):1\leq(1-\alpha)\beta p^{pool2}.$$

Next, we consider the high-quality firm. Since $p^{pool2}>1$, the high-quality firm's equilibrium profit is $\pi_h^{pool2}=\beta p^{pool2}$ by selling to all *quality-sensitive* consumers. We check all possible deviations by discussing the following cases. First, if the high-quality firm deviates to a σ' with $w(\sigma')=0$ and $p(\sigma')>p^{pool2}$, then the best deviation should be $p(\sigma')=1+d$, leading to a deviation profit of $\pi_h(\sigma')=\beta\alpha(1+d)$ by capturing only *informed quality-sensitive* consumers. Second, the high-quality firm will not deviate to any σ' with $w(\sigma')=0$ and $p(\sigma')\in(1,p^{pool2})$, because the deviation profit $\pi_h(\sigma')=\beta\alpha p(\sigma')$ must be lower than the equilibrium profit π_h^{pool2}. Third, if the high-quality firm deviates to a σ' with $w(\sigma')=0$ and $p(\sigma')=1$, then the deviation profit is $\pi_h(\sigma')=1$. Lastly, if the high-quality firm deviates to a σ' with $w(\sigma')=1$, the optimal deviation should satisfy $P(\sigma',S,I)=1+d$, $P(\sigma',N,I)=1$, and $P(\sigma',S,U)=P(\sigma',N,U)=1$, leading to a deviation profit of $\pi_h(\sigma')=\beta\alpha(1+d)+(1-\beta+\beta(1-\alpha))-k$. Summarizing the above cases, the high-quality firm's IC constraint is given by

$$(IC_h^{pool2}):\max\{\beta\alpha(1+d)+(1-\beta+\beta(1-\alpha))-k,1\}\leq\beta p^{pool2}.$$

[2] In the special $pool2$-type equilibrium where $w_h^{pool2}=w_l^{pool2}=0$ and $p_h^{pool2}=p_l^{pool2}=1$, we have $\pi_h^{pool2}=\pi_l^{pool2}=1$. To sustain this equilibrium, we need $1\geq 1+\beta\alpha d-k$ such that the high-quality firm will not deviate to a σ' with $w(\sigma')=1$, $P(\sigma',S,I)=1+d$, and $P(\sigma',N,I)=P(\sigma',S,U)=P(\sigma',N,U)=1$. So, this equilibrium exists when $k\geq\beta\alpha d$. Then, it is straightforward to show that whenever this $pool2$-type equilibrium exists, the high-type-best $sep1$-type equilibrium also exists and dominates this $pool2$-type equilibrium. Therefore, we consider $p^{pool2}>1$.

Step 2. We find the high-type-best PBE $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$ by accounting for both (IC_l^{pool2}) and (IC_h^{pool2}). Putting the IC constraints together, we have $\max\{\frac{1+\beta\alpha d-k}{\beta}, \frac{1}{(1-\alpha)\beta}\} \leq p^{pool2} \leq 1+\lambda d$. So, the $pool2$-type equilibrium exists when $\beta \geq \frac{1}{(1-\alpha)(1+\lambda d)}$ and $k \geq 1+\beta\alpha d-\beta(1+\lambda d)$. Furthermore, the one with $p^{pool2} = 1+\lambda d$ yields the highest profit for the high-quality firm. Thus, the high-type-best $pool2$-type equilibrium satisfies $\sigma_h^{pool2*} = (w_h^{pool2*} = 0, p_h^{pool2*} = 1+\lambda d)$ and $\sigma_l^{pool2*} = (w_l^{pool2*} = 0, p_l^{pool2*} = 1+\lambda d)$. In this high-type-best equilibrium, $\pi_h^{pool2*} = \beta(1+\lambda d)$ and $\pi_l^{pool2*} = (1-\alpha)\beta(1+\lambda d)$.

Step (iii): Find the high-type-best equilibrium for Case 3 $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$

Let $(\sigma_h^{sep3}, \sigma_l^{sep3}, \phi)$ be an $sep3$-type PBE in Case 3, where $w_h^{sep3} = 1$ and $w_l^{sep3} = 0$, hence $\sigma_h^{sep3} \neq \sigma_l^{sep3}$. We find $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$ in two steps. In step one, we derive the IC constraints for both firm types, which must be satisfied in equilibrium. In step two, we find $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$—the $sep3$-type PBE that yields the highest profit for the high-quality firm and satisfies the IC constraints.

Step 1. First, one can readily show that in an $sep3$-type equilibrium, the high-quality firm must charge $P_h^{sep3}(S,I) = 1+d$ for *informed quality-sensitive* consumers, $P_h^{sep3}(N,I) = 1$ for *informed quality-insensitive* consumers, and $P_h^{sep3}(N,U) = 1$ for *uninformed quality-insensitive* consumers. For *uninformed quality-sensitive* consumers, the high-quality firm charges $P_h^{sep3}(S,U)$. In an $sep3$-type equilibrium, it must also hold that for the low-quality firm, $w_l^{sep3} = 0$ and $p_l^{sep3} = 1$, which leads to $\pi_l^{sep3} = 1$.

We derive the low-quality firm's IC constraint. We check all possible deviations by discussing the following three cases. First, the low-quality firm will not deviate to a σ' with $w(\sigma') = 0$ and $p_l^{sep3} \neq 1$, because the deviation profit must be lower than $\pi_l^{sep3} = 1$. Second, the low-quality firm will not deviate to a σ' with $w(\sigma') = 1$ and $P(\sigma', S, U) \neq P_h^{sep3}(S,U)$ for *uninformed quality-sensitive* consumers. Under this deviation, *uninformed quality-sensitive* consumers will believe that the product has low quality with probability one. Hence, its deviation profit must be lower than $\pi_l^{sep3} = 1$ because of the adoption cost k. Third, we consider a deviation σ' with $w(\sigma') = 1$ and $P(\sigma', S, U) = P_h^{sep3}(S,U)$ for *uninformed quality-sensitive* consumers. For other types of consumers, the best deviation satisfies $P(\sigma', S, I) = P(\sigma', N, I) = P(\sigma', N, U) = 1$, thereby the deviation profit $\pi_l(\sigma') = (1-\beta+\beta\alpha) + \beta(1-\alpha)P_h^{sep3}(S,U) - k$. In this case, the low-quality firm will not deviate if and only if $(1-\beta+\beta\alpha) + \beta(1-\alpha)P_h^{sep3}(S,U) - k \leq 1$. Summarizing these cases, the low-quality firm's IC constraint is given by

$$(IC_l^{sep3}) : (1-\beta+\beta\alpha) + \beta(1-\alpha)P_h^{sep3}(S,U) - k \leq 1.$$

Next, we derive the high-quality firm's IC constraint. According to the personalized prices noted above, in an $sep3$-type equilibrium, the high-quality firm's profit is $\pi_h^{sep3} = \beta\alpha(1+d) + \beta(1-\alpha)P_h^{sep3}(S,U) + (1-\beta) - k$, where $P_h^{sep3}(S,U) \leq 1+d$. We check all possible deviations by discussing the following cases. First, the high-quality firm will not deviate to a σ' with $w(\sigma') = 1$ and $P(\sigma', S, U) \neq P_h^{sep3}(S,U)$. Under this deviation, *uninformed quality-sensitive* consumers will believe that the product has low quality with probability one. Hence, the high-quality firm cannot earn higher profit from these *uninformed quality-sensitive* consumers. Second, we consider a deviation σ' with $w(\sigma') = 0$ and $p(\sigma') = 1$, where the deviation profit is $\pi_h(\sigma') = 1$. Third, we consider a deviation σ' with $w(\sigma') = 0$ and $p(\sigma') = 1+d$, which is the optimal deviation among those deviations with $w(\sigma') = 0$ and $p(\sigma') > 1$ because only *informed quality-sensitive* consumers will purchase. Thus, the deviation profit is $\pi_h(\sigma') = \beta\alpha(1+d)$, which must be lower than π_h^{sep3} since $k < 1-\beta$. Summarizing the above cases, the high-quality firm's IC constraint is given by

$$(IC_h^{sep3}) : 1 \leq \beta\alpha(1+d) + \beta(1-\alpha)P_h^{sep3}(S,U) + (1-\beta) - k.$$

Step 2. We find the high-type-best PBE $(\sigma_l^{sep3*}, \sigma_h^{sep3*})$ by accounting for both (IC_l^{sep3}) and (IC_h^{sep3}). Putting the IC constraints together, we have $P_h^{sep3}(S,U) \in [1 - \frac{\alpha d}{1-\alpha} + \frac{k}{\beta(1-\alpha)}, \min\{1 + \frac{k}{\beta(1-\alpha)}, 1+d\}]$. Note that this set of $P_h^{sep3}(S,U)$ is non-empty given equation (3): $k < 1-\beta$ and $d > \frac{1-\beta}{\beta}$. Thus, among all possible PBEs, the one with $P_h^{sep3}(S,U) = \min\{1 + \frac{k}{\beta(1-\alpha)}, 1+d\}$ yields the highest profit for the high-quality firm.

To sum up, the high-type-best $sep3$-type equilibrium satisfies $\sigma_l^{sep3*} = (w_l^{sep3*} = 0, p_l^{sep3*} = 1)$ and $\sigma_h^{sep3*} = (w_h^{sep3*} = 1, P_h^{sep3*}(S,I) = 1+d, P_h^{sep3*}(S,U) = \min\{1 + \frac{k}{\beta(1-\alpha)}, 1+d\}, P_h^{sep3*}(N,I) = 1, P_h^{sep3*}(N,U) = 1)$. In this high-type-best equilibrium, $\pi_h^{sep3*} = \beta\alpha(1+d) + (1-\beta) + \beta(1-\alpha)\min\{1 + \frac{k}{\beta(1-\alpha)}, 1+d\} - k$ and $\pi_l^{sep3*} = 1$.

Step (iv): Find the high-type-best equilibrium for Case 4 $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$

Let $(\sigma_h^{sep4}, \sigma_l^{sep4}, \phi)$ be an $sep4$-type PBE in Case 4, where $w_h^{sep4} = w_l^{sep4} = 1$ and $\sigma_h^{sep4} \neq \sigma_l^{sep4}$. We find $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ in four steps. In step one, we show the equilibrium personalized prices for all consumers except for *uninformed quality-sensitive* consumers. In step two, we show that an $sep4$-type equilibrium must be *individually pooling* for *uninformed quality-sensitive* consumers: $P_h^{sep4}(S,U) = P_l^{sep4}(S,U)$. In step three, we derive the IC constraints for both firm types, which must be satisfied in equilibrium. In step four, we find $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$—the $sep4$-type PBE that yields the highest profit for the high-quality firm and satisfies the IC constraints.

Step 1. First of all, note that in an $sep4$-type equilibrium, the high-quality firm must charge personalized prices $P_h^{sep4}(S,I) = 1+d$ for *informed quality-sensitive* consumers, $P_h^{sep4}(N,I) = 1$ for *informed quality-insensitive* consumers, and $P_h^{sep4}(N,U) = 1$ for *uninformed quality-insensitive* consumers. Also, in an $sep4$-type equilibrium, the low-quality firm must charge personalized prices $P_l^{sep4}(S,I) = P_l^{sep4}(N,I) = P_l^{sep4}(N,U) = 1$.

Step 2. We show that an $sep4$-type equilibrium must satisfy $P_h^{sep4}(S,U) = P_l^{sep4}(S,U)$ for *uninformed quality-sensitive* consumers, under which they cannot tell the firm's quality type. Suppose not, then *uninformed quality-sensitive* consumers can tell the firm's quality type from the price, hence $P_l^{sep4}(S,U) \leq 1$, and the low-quality firm's profit is at most $1-k$. The low-quality firm hence can profitably deviate to charging a uniform price of one with profit one. Therefore, $P_h^{sep4}(S,U) = P_l^{sep4}(S,U) \leq 1 + \lambda d$.

<u>Step 3.</u> In equilibrium, the high-quality firm's profit is $\pi_h^{sep4} = \beta\alpha(1+d) + \beta(1-\alpha)P_h^{sep4}(S,U) + (1-\beta) - k$, while the low-quality firm's profit is $\pi_l^{sep4} = (1-\beta+\beta\alpha) + \beta(1-\alpha)P_l^{sep4}(S,U) - k$. Next, we check all possible deviations for the high-quality and low-quality firm, respectively.

Consider the low-quality firm. First, the low-quality firm will not deviate to a σ' with $w(\sigma') = 1$ and $P(\sigma', S, U) \neq P_l^{sep4}(S,U)$. Under this deviation, *uninformed quality-sensitive* consumers will believe that the product has low quality with probability one. Hence, the low-quality firm cannot earn higher profit from these *uninformed quality-sensitive* consumers. Second, we consider a deviation σ' with $w(\sigma') = 0$. Under this deviation, the optimal deviation profit is one. In this case, the low-quality firm will not deviate if and only if $1 \leq \pi_l^{sep4}$. Therefore, the low-quality firm's IC constraint is given by

$$(IC_l^{sep4}) : 1 \leq (1-\beta+\beta\alpha) + \beta(1-\alpha)P_l^{sep4}(S,U) - k.$$

Consider the high-quality firm. First, as above, the high-quality firm will not deviate to a σ' with σ_h^{sep4} except $P(\sigma', S, U) \neq P_h^{sep4}(S,U)$. Second, we consider a deviation σ' with $w(\sigma') = 0$. Under this deviation, all uninformed consumers believe that the product has low quality with probability one. Thus, the optimal deviation profit will be $\max\{1, \beta\alpha(1+d)\}$. Since $k < 1-\beta$, $\beta\alpha(1+d) < \pi_h^{sep4}$. Therefore, the high-quality firm's IC constraint is given by

$$(IC_h^{sep4}) : 1 \leq \beta\alpha(1+d) + \beta(1-\alpha)P_h^{sep4}(S,U) + (1-\beta) - k.$$

<u>Step 4.</u> Putting the IC constraints together, we have $1 + \frac{k}{\beta(1-\alpha)} \leq P_h^{sep4}(S,U) = P_l^{sep4}(S,U) \leq 1 + \lambda d$. Thus, the $sep4$-type PBE exists when $k \leq \beta(1-\alpha)\lambda d$. Moreover, $P_h^{sep4}(S,U) = P_l^{sep4}(S,U) = 1 + \lambda d$ yields the highest profit for the high-quality firm. Hence, the high-type-best $sep4$-type equilibrium satisfies $\sigma_h^{sep4*} = (w_h^{sep4*} = 1, P_h^{sep4*}(S,I) = 1+d, P_h^{sep4*}(N,I) = 1, P_h^{sep4*}(S,U) = 1+\lambda d, P_h^{sep4*}(N,U) = 1)$ and $\sigma_l^{sep4*} = (w_l^{sep4*} = 1, P_l^{sep4*}(S,I) = 1, P_l^{sep4*}(N,I) = 1, P_l^{sep4*}(S,U) = 1+\lambda d, P_l^{sep4*}(N,U) = 1)$. In this high-type-best equilibrium, $\pi_h^{sep4*} = \beta\alpha(1+d) + \beta(1-\alpha)(1+\lambda d) + (1-\beta) - k$ and $\pi_l^{sep4*} = (1-\beta+\beta\alpha) + \beta(1-\alpha)(1+\lambda d) - k$.

Deriving the SUE across parameters

Step (v): Derive the SUE

In this step, we compare $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$, $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$, $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$, and $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ to derive the SUE that yields the highest profit for the high-quality firm. This process will yield a unique equilibrium. We will first show the results concerning the parameter β, which then can be translated regarding the parameters d and α, respectively. Table A2 summarizes the equilibrium outcomes. Lastly, we show the results concerning the parameter k, which is the cost of adopting the PP technology.

(i) The SUE concerning β. We divide the discussion into three β regions: $\beta \leq \frac{k}{(1-\alpha)d}$, $\frac{k}{(1-\alpha)d} < \beta < \frac{k}{(1-\alpha)\lambda d}$, and $\beta \geq \frac{k}{(1-\alpha)\lambda d}$.

Region 1: $\beta \leq \frac{k}{(1-\alpha)d}$. In this region, the $sep4$-type PBE does not exist. Moreover, $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$ must be dominated by $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$, because $\pi_h^{sep3*} = \beta(1+d) + (1-\beta) - k > \pi_h^{pool2*} = \beta(1+\lambda d)$. Similarly, $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ must be dominated by $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$, because $\pi_h^{sep3*} = \beta(1+d)+(1-\beta)-k > \pi_h^{sep1*} = \beta(1+d)$. Therefore, in Region 1, the SUE is $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$.

Region 2: $\frac{k}{(1-\alpha)d} < \beta < \frac{k}{(1-\alpha)\lambda d}$. In this region, note that the $sep4$-type PBE still does not exist. Moreover, $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$ must be dominated by $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$, because $\pi_h^{sep3*} = \beta\alpha(1+d)+\beta(1-\alpha)+k+(1-\beta)-k > \beta\alpha(1+d)+\beta(1-\alpha)(1+\lambda d)+(1-\beta)-k > \pi_h^{pool2*} = \beta(1+\lambda d)$. Thus, we are left to compare between $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ and $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$. In Region 2, we have $\pi_h^{sep3*} = 1+\beta\alpha d$ and $\pi_h^{sep1*} = \min\{\beta(1+d), \frac{1}{1-\alpha}\}$. Accounting for the existence condition of $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$, $k \geq 1+\beta\alpha d - \min\{\frac{1}{(1-\alpha)}, \beta(1+d)\}$, we divide the discussion into two cases.

First, when $\frac{1-k}{1+(1-\alpha)d} \leq \beta \leq \frac{1}{(1-\alpha)(1+d)}$, $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ exists and $\pi_h^{sep1*} = \beta(1+d)$, so $\pi_h^{sep1*} > \pi_h^{sep3*}$ leads to $\beta > \frac{1}{1+(1-\alpha)d}$. Second, when $\frac{1}{(1-\alpha)(1+d)} < \beta \leq \frac{\alpha+(1-\alpha)k}{(1-\alpha)\alpha d}$, $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ exists and $\pi_h^{sep1*} = \frac{1}{1-\alpha}$, so $\pi_h^{sep1*} > \pi_h^{sep3*}$ leads to $\beta < \frac{1}{(1-\alpha)d}$. Combining these two cases, $\pi_h^{sep1*} > \pi_h^{sep3*}$ when $\frac{1}{1+(1-\alpha)d} < \beta < \frac{1}{(1-\alpha)d}$ (which is a subset of the existence condition of $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$).

To summarize, in Region 2, the SUE is $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ if and only if $\beta \in \mathcal{P}_{R2}$, where

$$\mathcal{P}_{R2} = \begin{cases} \emptyset & \text{if } k \leq \frac{(1-\alpha)\lambda d}{1+(1-\alpha)d}, \\ \left(\frac{1}{1+(1-\alpha)d}, \min\left\{\frac{1}{(1-\alpha)d}, \frac{k}{(1-\alpha)\lambda d}\right\}\right) & \text{if } \frac{(1-\alpha)\lambda d}{1+(1-\alpha)d} < k \leq \frac{(1-\alpha)d}{1+(1-\alpha)d}, \\ \left(\frac{k}{(1-\alpha)d}, \min\left\{\frac{1}{(1-\alpha)d}, \frac{k}{(1-\alpha)\lambda d}\right\}\right) & \text{if } k > \frac{(1-\alpha)d}{1+(1-\alpha)d}. \end{cases}$$

Note that $\mathcal{P}_{R2}$ may be empty, denoted as $\emptyset$.

Region 3: $\beta \geq \frac{k}{(1-\alpha)\lambda d}$. In this region, the $sep4$-type PBE exists. Moreover, $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$ is dominated by $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$, because $\pi_h^{sep4*} = 1+\beta\alpha d+\beta(1-\alpha)\lambda d - k \geq \pi_h^{sep3*} = 1+\beta\alpha d$. So we are left to compare Cases 1, 2, and 4. We next divide the discussion into the following three cases:

When $\beta \geq \frac{1}{(1-\alpha)(1+\lambda d)}$, $\pi_h^{sep4*} = \beta\alpha(1+d)+\beta(1-\alpha)(1+\lambda d)+(1-\beta)-k > \pi_h^{pool2*} = \beta(1+\lambda d) \geq \pi_h^{sep1*} = \frac{1}{1-\alpha}$, so the SUE is $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ in this case.

When $\frac{1}{(1-\alpha)(1+d)} < \beta < \frac{1}{(1-\alpha)(1+\lambda d)}$, $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$ does not exist. $\pi_h^{sep1*} = \frac{1}{1-\alpha} > \pi_h^{sep4*}$ leads to $\beta < \frac{\alpha+(1-\alpha)k}{(1-\alpha)\alpha d+(1-\alpha)^2\lambda d}$; note that under this condition, $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ always exists.[3]

When $\beta \leq \frac{1}{(1-\alpha)(1+d)}$, $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$ does not exist. $\pi_h^{sep1*} = \beta(1+d) > \pi_h^{sep4*}$ leads to $\beta > \frac{1-k}{1+(1-\alpha)(1-\lambda)d}$; note that under this condition, $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ always exists.

[3] One can readily see $\pi_h^{sep4*} > 1+\beta\alpha d - k$, so if the derived condition holds (i.e., $\pi_h^{sep1*} > \pi_h^{sep4*}$), then the existence condition for $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ will also hold (i.e., $\pi_h^{sep1*} > 1+\beta\alpha d - k$).

To sum up, in Region 3, the SUE is $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ if and only if $\beta \in \mathcal{P}_{R3}$, where

$$\mathcal{P}_{R3} = \begin{cases} \emptyset & \text{if } k \le \frac{(1-\alpha)\lambda d-\alpha}{(1-\alpha)(1+d)}, \\ \left(\frac{1-k}{1+(1-\alpha)(1-\lambda)d}, \frac{\alpha+(1-\alpha)k}{(1-\alpha)\alpha d+(1-\alpha)^2\lambda d}\right) & \text{if } \frac{(1-\alpha)\lambda d-\alpha}{(1-\alpha)(1+d)} < k \le \frac{(1-\alpha)\lambda d}{1+(1-\alpha)d}, \\ \left(\frac{k}{(1-\alpha)\lambda d}, \frac{\alpha+(1-\alpha)k}{(1-\alpha)\alpha d+(1-\alpha)^2\lambda d}\right) & \text{if } \frac{(1-\alpha)\lambda d}{1+(1-\alpha)d} < k < \lambda, \\ \emptyset & \text{if } k \ge \lambda. \end{cases}$$

<u>Merging Regions 1-3.</u> We further combine Region 2 ($\mathcal{P}_{R2}$) and Region 3 ($\mathcal{P}_{R3}$) to show that the SUE is $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ when β is intermediate, i.e., $\beta \in (\beta_1, \beta_2)$.

When $k \le \frac{(1-\alpha)\lambda d-\alpha}{(1-\alpha)(1+d)}$, both $\mathcal{P}_{R2}$ and $\mathcal{P}_{R3}$ are empty, so $(\beta_1, \beta_2) = \emptyset$. When $\frac{(1-\alpha)\lambda d-\alpha}{(1-\alpha)(1+d)} < k \le \frac{(1-\alpha)\lambda d}{1+(1-\alpha)d}$, $\mathcal{P}_{R2}$ is empty, so the condition for the SUE being $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ is $\beta \in (\beta_1, \beta_2) = \mathcal{P}_{R3}$. When $\frac{(1-\alpha)\lambda d}{1+(1-\alpha)d} < k < \lambda$, neither $\mathcal{P}_{R2}$ nor $\mathcal{P}_{R3}$ is empty. In this case, $\mathcal{P}_{R2}$ and $\mathcal{P}_{R3}$ can be merged into a continuous region, because $\pi_h^{sep3*} = \pi_h^{sep4*}$ at $\beta = \frac{k}{(1-\alpha)\lambda d}$; the condition for the SUE being $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ is $\beta \in (\beta_1, \beta_2) = \mathcal{P}_{R4}$, where $\mathcal{P}_{R4} = \left(\max\left\{\frac{1}{1+(1-\alpha)d}, \frac{k}{(1-\alpha)d}\right\}, \frac{\alpha+(1-\alpha)k}{(1-\alpha)\alpha d+(1-\alpha)^2\lambda d}\right)$. When $k \ge \lambda$, $\mathcal{P}_{R3}$ is empty, so the condition for the SUE being $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ is $\beta \in (\beta_1, \beta_2) = \mathcal{P}_{R2}$.

Thus, we conclude that the SUE is $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ if and only if $\beta \in (\beta_1, \beta_2)$, where β_1 (β_2) is the lower (upper) boundary in the aforementioned β intervals $\mathcal{P}_{R2}$, $\mathcal{P}_{R3}$, or $\mathcal{P}_{R4}$. Note that (β_1, β_2) may be empty as previously discussed.

Let $\hat{\beta} \equiv \frac{k}{(1-\alpha)\lambda d}$, the β threshold above which $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ exists. The above analysis shows that $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$ can never be the SUE, so we only need to consider which of Cases 1, 3, or 4 is the SUE. We have shown that $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ is the SUE when $\beta \in (\beta_1, \beta_2)$. We can also show that when $\beta \notin (\beta_1, \beta_2)$, the SUE is $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$ when $\beta < \hat{\beta}$ and is $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ when $\beta \ge \hat{\beta}$. This is because outside the region (β_1, β_2), the SUE is either $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$ or $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$, depending on whether $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ exists (i.e., whether $\beta \ge \hat{\beta}$), because whenever $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ exists, it will dominate $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$ (see the above analysis in Regions 2 and 3).

We summarize the equilibrium outcomes in Table A2. When the SUE is $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$, we refer to it as the *no-adoption* equilibrium, because both firm types do not adopt the PP technology; when the SUE is $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$, we refer to it as the *partial-adoption* equilibrium, because only the high-quality firm adopts the PP technology; when the SUE is $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$, we refer to it as the *universal-adoption* equilibrium, because both firm types adopt the PP technology.

(ii) The SUE concerning d. We can translate the equilibrium results into equivalent ones with respect to d. Following the above proof, we can divide the discussion into three equivalent d regions: $d \le \frac{k}{(1-\alpha)\beta}$, $\frac{k}{(1-\alpha)\beta} < d < \frac{k}{(1-\alpha)\lambda\beta}$, and $d \ge \frac{k}{(1-\alpha)\lambda\beta}$.

Table A2. Equilibrium outcomes for the main model

Conditions	Equilibrium types	Equilibrium prices
$\beta \in (\beta_1, \beta_2)$ or $d \in (d_1, d_2)$ or $\alpha \in (\alpha_1, \alpha_2)$	No adoption $w_h^* = w_l^* = 0$	$p_h^* = \min\{1+d, \frac{1}{(1-\alpha)\beta}\}$ $p_l^* = 1$
$\beta \in (0, \hat{\beta}) \setminus (\beta_1, \beta_2)$ or $d \in (0, \hat{d}) \setminus (d_1, d_2)$ or $\alpha \in (\hat{\alpha}, 1) \setminus (\alpha_1, \alpha_2)$	Partial adoption $w_h^* = 1, w_l^* = 0$	$P_h^*(S,I) = 1+d, P_h^*(S,U) = \min\{1+\frac{k}{\beta(1-\alpha)}, 1+d\}, P_h^*(N,I) = 1, P_h^*(N,U) = 1$ $p_l^* = 1$
$\beta \in [\hat{\beta}, 1) \setminus (\beta_1, \beta_2)$ or $d \in [\hat{d}, \infty) \setminus (d_1, d_2)$ or $\alpha \in (0, \hat{\alpha}] \setminus (\alpha_1, \alpha_2)$	Universal adoption $w_h^* = w_l^* = 1$	$P_h^*(S,I) = 1+d, P_h^*(N,I) = 1, P_h^*(S,U) = 1+\lambda d, P_h^*(N,U) = 1$ $P_l^*(S,I) = 1, P_l^*(N,I) = 1, P_l^*(S,U) = 1+\lambda d, P_l^*(N,U) = 1$

In Region 1 ($d \leq \frac{k}{(1-\alpha)\beta}$), the SUE is $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$. In Region 2 ($\frac{k}{(1-\alpha)\beta} < d < \frac{k}{(1-\alpha)\lambda\beta}$), the SUE is $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ if and only if $d \in \mathcal{Q}_{R2}$, where

$$\mathcal{Q}_{R2} = \begin{cases} \emptyset & \text{if } k \leq (1-\beta)\lambda, \\ \left(\frac{1-\beta}{(1-\alpha)\beta}, \frac{k}{(1-\alpha)\lambda\beta}\right) & \text{if } (1-\beta)\lambda < k \leq \lambda, \\ \left(\frac{1-\beta}{(1-\alpha)\beta}, \frac{1}{(1-\alpha)\beta}\right) & \text{if } k > \lambda. \end{cases}$$

In Region 3 ($d \geq \frac{k}{(1-\alpha)\lambda\beta}$), the SUE is $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ if and only if $d \in \mathcal{Q}_{R3}$, where

$$\mathcal{Q}_{R3} = \begin{cases} \emptyset & \text{if } k \leq (1-\beta)\lambda - (1-\lambda)\alpha\beta, \\ \left(\frac{1-\beta-k}{(1-\alpha)(1-\lambda)\beta}, \frac{\alpha+(1-\alpha)k}{(1-\alpha)\alpha\beta+(1-\alpha)^2\lambda\beta}\right) & \text{if } (1-\beta)\lambda - (1-\lambda)\alpha\beta < k \leq (1-\beta)\lambda, \\ \left(\frac{k}{(1-\alpha)\lambda\beta}, \frac{\alpha+(1-\alpha)k}{(1-\alpha)\alpha\beta+(1-\alpha)^2\lambda\beta}\right) & \text{if } (1-\beta)\lambda < k < \lambda, \\ \emptyset & \text{if } k \geq \lambda. \end{cases}$$

Similarly, we can combine $\mathcal{Q}_{R2}$ and $\mathcal{Q}_{R3}$ to show that the SUE is $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ if and only if $d \in (d_1, d_2)$, which may be empty. Let $\hat{d} \equiv \frac{k}{(1-\alpha)\lambda\beta}$. Then, outside the region (d_1, d_2), the SUE is $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$ when $d < \hat{d}$ and is $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ when $d \geq \hat{d}$.

(iii) The SUE concerning α. We can translate the equilibrium results into equivalent ones with respect to α. Following the above proof, we can divide the discussion into three equivalent α regions: $\alpha \geq \frac{\beta d - k}{\beta d}$, $\frac{\beta\lambda d - k}{\beta\lambda d} < \alpha < \frac{\beta d - k}{\beta d}$, and $\alpha \leq \frac{\beta\lambda d - k}{\beta\lambda d}$.

In Region 1 ($\alpha \geq \frac{\beta d - k}{\beta d}$), the SUE is $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$. In Region 2 ($\frac{\beta\lambda d - k}{\beta\lambda d} < \alpha < \frac{\beta d - k}{\beta d}$), the SUE is $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ if and only if $\alpha \in \mathcal{O}_{R2}$, where

$$\mathcal{O}_{R2} = \begin{cases} \emptyset & \text{if } k \leq (1-\beta)\lambda, \\ \left(\frac{\beta\lambda d - k}{\beta\lambda d}, \frac{\beta(1+d)-1}{\beta d}\right) & \text{if } (1-\beta)\lambda < k \leq \lambda, \\ \left(\frac{\beta d - 1}{\beta d}, \frac{\beta(1+d)-1}{\beta d}\right) & \text{if } k > \lambda. \end{cases}$$

In Region 3 ($\alpha \leq \frac{\beta\lambda d-k}{\beta\lambda d}$), the SUE is $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ if and only if $\alpha \in \mathcal{O}_{R3}$, where

$$\mathcal{O}_{R3} = \begin{cases} \emptyset & \text{if } k \leq \frac{1-\beta-\beta d+\lambda d}{1+d}, \\ \left(\tilde{\alpha}, \frac{\beta d(1-\lambda)-(1-k-\beta)}{\beta d(1-\lambda)}\right) & \text{if } \frac{1-\beta-\beta d+\lambda d}{1+d} < k \leq (1-\beta)\lambda, \\ \left(\tilde{\alpha}, \frac{\beta\lambda d-k}{\beta\lambda d}\right) & \text{if } (1-\beta)\lambda < k < \lambda, \\ \emptyset & \text{if } k \geq \lambda, \end{cases}$$

where $\tilde{\alpha} = \frac{k-1+\beta d(1-2\lambda)+\sqrt{\beta^2 d^2+(1-k)^2-2\beta d(1+k-2\lambda)}}{2\beta d(1-\lambda)}$ is solved from $\beta = \frac{\alpha+(1-\alpha)k}{(1-\alpha)\alpha d+(1-\alpha)^2\lambda d}$.

Similarly, we can combine $\mathcal{O}_{R2}$ and $\mathcal{O}_{R3}$ to show that the SUE is $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ if and only if $\alpha \in (\alpha_1, \alpha_2)$, which may be empty. Let $\hat{\alpha} \equiv \frac{\beta\lambda d-k}{\beta\lambda d}$. Then, outside the region (α_1, α_2), the SUE is $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$ when $\alpha > \hat{\alpha}$ and is $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ when $\alpha \leq \hat{\alpha}$.

(iv) The SUE concerning k. Following the above proof, we still divide the discussion into three equivalent k regions: $k \geq \beta(1-\alpha)d$, $\beta(1-\alpha)\lambda d < k < \beta(1-\alpha)d$, and $k \leq \beta(1-\alpha)\lambda d$.

In the region of $k \leq \beta(1-\alpha)\lambda d$, we only need to compare $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ with $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$. One can readily show that the SUE is $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ when $k > k_1 = 1+\beta\alpha d+\beta(1-\alpha)\lambda d-\min\{\beta(1+d), \frac{1}{1-\alpha}\}$, a case which can arise in this region only if $1+\beta\alpha d < \min\{\beta(1+d), \frac{1}{1-\alpha}\}$.

In the region of $\beta(1-\alpha)\lambda d < k < \beta(1-\alpha)d$, we only need to compare $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ with $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$. One can readily show that the SUE is $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ when $\pi_h^{sep3*} = 1+\beta\alpha d < \pi_h^{sep1*} = \min\{\beta(1+d), \frac{1}{1-\alpha}\}$ and $k \geq 1+\beta\alpha d - \min\{\frac{1}{1-\alpha}, \beta(1+d)\}$; the latter ensures the existence of $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$.

Summarizing the analysis of the above two regions, one can see that when $1+\beta\alpha d < \min\{\beta(1+d), \frac{1}{1-\alpha}\}$, the SUE is $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ when $k \leq k_1$ and $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ when $k > k_1$. Note that under the condition $1+\beta\alpha d < \min\{\beta(1+d), \frac{1}{1-\alpha}\}$, we do not have the region of $k \geq \beta(1-\alpha)d$ since we have constrained $k < 1-\beta$. Conversely, when $1+\beta\alpha d \geq \min\{\beta(1+d), \frac{1}{1-\alpha}\}$, $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ cannot be the SUE. Then, the SUE is $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ when $k \leq \beta(1-\alpha)\lambda d$ and $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$ otherwise.

OA.2. Equilibrium welfare impacts

Proof of Proposition 4. We prove the three parts in Proposition 4 in order. Recall from Table A2 that eventually there are three types of equilibrium with respect to α.

For part (i), it is easy to check that the high-quality firm's equilibrium profit is continuous and non-decreasing with respect to α. Moreover, if $\alpha_2 < \hat{\alpha}$, then the equilibrium transitions from *no-adoption* to *universal-adoption* as α increases passing α_2 (i.e., from $\alpha \to \alpha_2^-$ to $\alpha \to \alpha_2^+$). In this case, as α increases, we have $\lim_{\alpha\to\alpha_2^+} \pi_l^* = (1-\beta+\beta\alpha)+\beta(1-\alpha)(1+\lambda d)-k > \lim_{\alpha\to\alpha_2^-} \pi_l^* = 1$, which means that the low-quality firm's equilibrium profit can increase discontinuously at α_2.

For parts (ii) and (iii), we first derive the *ex-ante* consumer surplus and social welfare under each type of equilibrium, and then examine how the equilibrium consumer surplus and social welfare vary with α.

When $\alpha \in (\alpha_1, \alpha_2)$, the equilibrium is *no-adoption*. Then, the ex-ante consumer surplus is

$$CS^* = \lambda CS_h^* + (1-\lambda)CS_l^* = \begin{cases} \lambda \frac{(1+d)(1-\alpha)\beta - 1}{1-\alpha} & \text{if } \beta > \frac{1}{(1+d)(1-\alpha)} \\ 0 & \text{if } \beta \leq \frac{1}{(1+d)(1-\alpha)} \end{cases},$$

and the ex-ante social welfare is $SW^* = \lambda SW_h^* + (1-\lambda)SW_l^* = 1 + \beta d\lambda - (1-\beta)\lambda$.

When $\alpha \in (\hat{\alpha}, 1) \setminus (\alpha_1, \alpha_2)$, the equilibrium is *partial-adoption*, so the ex-ante consumer surplus is

$$CS^* = \lambda CS_h^* + (1-\lambda)CS_l^* = \begin{cases} (1-\alpha)\beta d\lambda - k\lambda & \text{if } k < (1-\alpha)\beta d \\ 0 & \text{if } k \geq (1-\alpha)\beta d \end{cases},$$

and the ex-ante social welfare is $SW^* = \lambda SW_h^* + (1-\lambda)SW_l^* = 1 + \beta d\lambda - k\lambda$.

When $\alpha \in (0, \hat{\alpha}) \setminus (\alpha_1, \alpha_2)$, the equilibrium is *universal-adoption*, so the ex-ante consumer surplus is

$$CS^* = \lambda CS_h^* + (1-\lambda)CS_l^* = 0,$$

and the ex-ante social welfare is $SW^* = \lambda SW_h^* + (1-\lambda)SW_l^* = 1 + \beta d\lambda - k$.

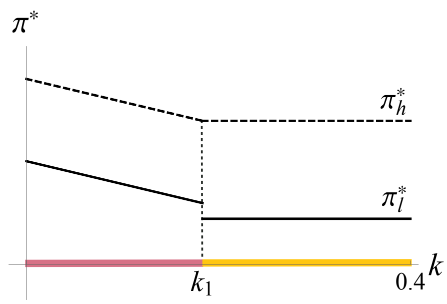
Then, it is easy to check that as α increases, CS^* weakly decreases and SW^* stays constant *within* each type of equilibrium. However, if $\alpha_1 < \hat{\alpha}$, then the equilibrium transitions from *universal-adoption* to *no-adoption* as α increases passing α_1 (i.e., from $\alpha \to \alpha_1^-$ to $\alpha \to \alpha_1^+$). In this case, one can show that as α increases, the equilibrium social welfare can decrease discontinuously at α_1, because $\lim_{\alpha \to \alpha_1^-} SW^* = 1 + \beta d\lambda - k > \lim_{\alpha \to \alpha_1^+} SW^* = 1 + \beta d\lambda - (1-\beta)\lambda$ when $k < (1-\beta)\lambda$. □

Proof of Corollary 1. We prove the two parts in Corollary 1 in order. Recall from Table A2 that eventually there are three types of equilibrium with respect to β or d.

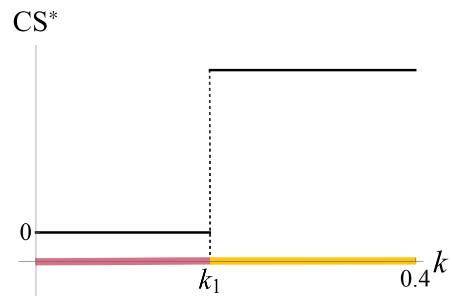
For part (i), it is easy to check that the high-quality firm's equilibrium profit is continuous and non-decreasing with respect to β and d. Also, the low-quality firm's equilibrium profit is non-decreasing with respect to β and d *within* the *no-adoption* and *partial-adoption* equilibria. However, the low-quality firm's equilibrium profit increases with β and d *within* the universal-adoption equilibrium. Moreover, if $\hat{\beta} < \beta_2$ ($\hat{d} < d_2$), then the equilibrium transitions from *no-adoption* to *universal-adoption* as β (d) increases passing β_2 (d_2), i.e., from $\beta \to \beta_2^-$ to $\beta \to \beta_2^+$ (from $d \to d_2^-$ to $d \to d_2^+$). In this case, as β or d increases, we have $\lim_{\beta \to \beta_2^+} \pi_l^* = (1-\beta+\beta\alpha) + \beta(1-\alpha)(1+\lambda d) - k > \lim_{\beta \to \beta_2^-} \pi_l^* = 1$ and $\lim_{d \to d_2^+} \pi_l^* = (1-\beta+\beta\alpha) + \beta(1-\alpha)(1+\lambda d) - k > \lim_{d \to d_2^-} \pi_l^* = 1$, which means that the low-quality firm's equilibrium profit can increase discontinuously at β_2 or d_2. One can also see that $\pi_l^* = (1-\beta+\beta\alpha) + \beta(1-\alpha)(1+\lambda d) - k$ increases with d (β) for $d > d_2$ ($\beta > \beta_2$) where the equilibrium is a universal-adoption one.

We have derived the equilibrium consumer surplus and social welfare in the proof of Proposition 4. One can see that CS^* and SW^* are non-decreasing with respect to β and d *within* each type of equilibrium. However, if $\hat{\beta} < \beta_2$ ($\hat{d} < d_2$), then the equilibrium transitions from *no-adoption* to *universal-adoption* as β (d) increases passing β_2 (d_2), i.e., from $\beta \to \beta_2^-$ to $\beta \to \beta_2^+$ (from $d \to d_2^-$ to $d \to d_2^+$). In this case, as β or d increases, the equilibrium consumer surplus can decrease discontinuously at β_2 or d_2, because $\lim_{\beta\to\beta_2^-} CS^* = \lambda\frac{(1+d)(1-\alpha)\beta-1}{1-\alpha} > \lim_{\beta\to\beta_2^+} CS^* = 0$ and $\lim_{d\to d_2^-} CS^* = \lambda\frac{(1+d)(1-\alpha)\beta-1}{1-\alpha} > \lim_{d\to d_2^+} CS^* = 0$. In addition, if $\hat{\beta} > \beta_1$ ($\hat{d} > d_1$), then the equilibrium transitions from *partial-adoption* to *no-adoption* as β (d) increases passing β_1 (d_1), i.e., from $\beta \to \beta_1^-$ to $\beta \to \beta_1^+$ (from $d \to d_1^-$ to $d \to d_1^+$). Therefore, as β or d increases, the equilibrium social welfare can decrease discontinuously at β_1 or d_1, because $\lim_{\beta\to\beta_1^-} SW^* = 1+\beta d\lambda - k\lambda > \lim_{\beta\to\beta_1^+} SW^* = 1+\beta d\lambda - (1-\beta)\lambda$ and $\lim_{d\to d_1^-} SW^* = 1+\beta d\lambda - k\lambda > \lim_{d\to d_1^+} SW^* = 1+\beta d\lambda - (1-\beta)\lambda$. □

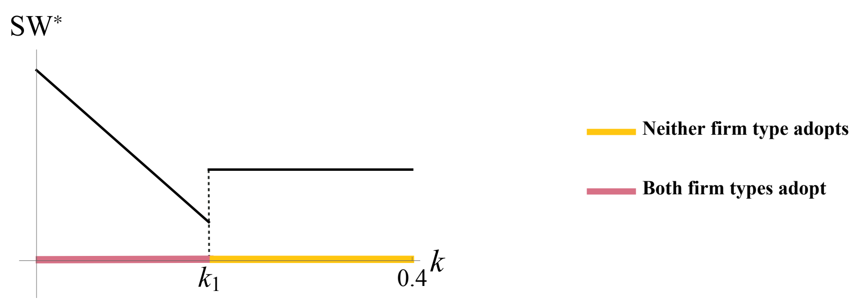
Figure A1. The impacts of PP adoption cost on the equilibrium outcomes



(a) Firm profit



(b) Consumer surplus



(c) Social welfare

Note. $d = 2$, $\beta = 0.6$, $\lambda = 0.3$, and $\alpha = 0.3$.

OA.3. Equilibrium refinement with the D1 criterion

We use the D1 criterion to refine the equilibrium. In particular, our analysis will reveal two points. (1) In our setting, the D1 criterion does not generally ensure a unique equilibrium in our model. (2) Our key insight that the high-quality firm may refrain from adopting PP still holds: in particular, the no-adoption equilibrium with $\sigma_h^* = (w_h^* = 0, p_h^* = \min\{\frac{1}{(1-\alpha)\beta}, 1+d\})$ and $\sigma_l^* = (w_l^* = 0, p_l^* = 1)$ can uniquely survive the D1 criterion. The second point proves Lemma 2.

Recall that in our setting consumers may not observe the firm's full pricing action σ. If the firm does not adopt the PP technology, consumer i observes $\gamma_i(\sigma) = (w(\sigma) = 0, p(\sigma))$, namely $w(\sigma) = 0$ together with the uniform price $p(\sigma)$; if the firm adopts the PP technology, she observes $\gamma_i(\sigma) = (w(\sigma) = 1, P(\sigma,\theta_i,\tau_i))$, namely $w(\sigma) = 1$ together with her own personalized price $P(\sigma,\theta_i,\tau_i)$—only a point of the firm's personalized pricing function $P(\sigma,\theta,\tau)$. An uninformed consumer forms her quality belief based on $\gamma_i(\sigma)$. As noted in the model setup, the willingness to pay of quality-insensitive consumers is independent of product quality, so we can focus on discussing the quality belief of uninformed quality-sensitive consumers; we denote their belief by $\phi(\gamma_i(\sigma))$, the probability they assign to the firm being high-quality based on their observed information $\gamma_i(\sigma)$. For convenience, when we refer to an off-equilibrium action in the subsequent analysis, we use ϕ' to denote the uninformed quality-sensitive consumers' potential out-of-equilibrium belief that the firm is high quality.

Note that we have identified all possible PBEs in the main analysis (OA.1), as we assume the pessimistic off-equilibrium beliefs: $\phi(\gamma_i(\sigma)) = 0$ if $\gamma_i(\sigma) \notin \{\gamma_i(\sigma_h), \gamma_i(\sigma_l)\}$. That is, any off-equilibrium strategy is believed to be associated with the low-quality firm. Focusing on pessimistic beliefs is without loss of generality for identifying all PBE outcomes. The reason is that each firm type's profit from a deviation is weakly increasing in the off-equilibrium belief, because uninformed quality-sensitive consumers' willingness to pay rises with the belief; pessimistic beliefs therefore minimize both firm types' deviation profits, slackening each type's incentive-compatibility constraint. Hence, if a PBE outcome can be supported by another off-equilibrium belief, then it can also be supported by our proposed pessimistic beliefs. However, the D1 criterion considers that, among all potential deviators, a message receiver should restrict the beliefs to only those types of message senders who most likely send an off-equilibrium message. In our model, by deviating to an off-equilibrium strategy, if the high-quality firm benefits from a larger set of out-of-equilibrium beliefs than the low-quality firm, then the out-of-equilibrium belief should be $\phi(\gamma_i(\sigma)) = 1$, rather than $\phi(\gamma_i(\sigma)) = 0$; in such a situation, the PBE supported by the out-of-equilibrium belief $\phi(\gamma_i(\sigma)) = 0$ is questionable and may fail the D1 criterion.

In the main analysis, we examine four cases to identify all possible PBEs (see Table A1 and OA.1). Below we revisit each case respectively and use the D1 criterion to refine these identified PBEs. Then, we provide a summary. Without loss of generality, we restrict attention to deviations that satisfy the purchase constraint for uninformed quality-sensitive consumers (i.e., $p' \le 1 + \phi' d$ when $w' = 0$ or $P'(S,U) \le 1 + \phi' d$ when $w' = 1$) because otherwise neither firm type will deviate based on our previous identification of all possible PBEs.

Refine all PBEs in Case 1 under the D1 criterion: In Case 1, an $sep1$-type PBE has $w_h^{sep1} = w_l^{sep1} = 0$ and $p_h^{sep1} \neq p_l^{sep1}$. Recall that we have identified all $sep1$-type PBEs based on $IC_l^{sep1} : (1-\alpha)\beta p_h^{sep1} \le 1$ and $IC_h^{sep1} : \max\{1, \beta\alpha(1+d) + (1-\beta+\beta(1-\alpha)) - k\} \le \beta p_h^{sep1}$. The firm's equilibrium profit is $\pi_h^{sep1} = \beta p_h^{sep1}$ and $\pi_l^{sep1} = 1$. In particular, the high-type-best $sep1$-type PBE satisfies $\sigma_h^{sep1*} = (w_h^{sep1*} = 0, p_h^{sep1*} = \min\{\frac{1}{(1-\alpha)\beta}, 1+d\})$ and $\sigma_l^{sep1*} = (w_l^{sep1*} = 0, p_l^{sep1*} = 1)$.

First, we show that all $sep1$-type PBEs except the high-type-best $sep1$-type PBE fail the D1 criterion. Suppose there exists an $sep1$-type PBE with $p_h^{sep1} < p_h^{sep1*}$ (which implies that the high-type-best PBE with p_h^{sep1*} also exists). Consider an off-equilibrium action with $w' = 0$ and $p' = p_h^{sep1} + \epsilon$, where $\epsilon > 0$ is sufficiently small. Then, under any belief ϕ', the low-quality firm is not willing to deviate as $(1-\alpha)\beta p' \leq 1$ (which holds since p_h^{sep1*} satisfies IC_l^{sep1}). However, the high-quality firm can profitably deviate to p' with $\phi' = 1$. In other words, the high-quality firm benefits from a larger set of out-of-equilibrium beliefs than the low-quality firm. Thus, any $sep1$-type PBE with $p_h^{sep1} < p_h^{sep1*}$ fails the D1 criterion.

Next, we check whether the high-type-best $sep1$-type PBE survives the D1 criterion. First, consider an off-equilibrium action with $w' = 0$ and $p' > p_h^{sep1*}$ (clearly neither firm type will deviate to $p' < p_h^{sep1*}$). Note that this deviation check only applies when the high-type-best $sep1$-type PBE has $p_h^{sep1*} = \frac{1}{(1-\alpha)\beta} < 1+d$. In this case, for $p' \leq 1+\phi' d$, the low-quality firm is willing to deviate when $(1-\alpha)\beta p' > 1 (\Leftrightarrow p' > \frac{1}{\beta(1-\alpha)})$, while the high-quality firm is willing to deviate when $\beta p' > \frac{1}{(1-\alpha)} (\Leftrightarrow p' > \frac{1}{\beta(1-\alpha)})$. Thus, for such an off-equilibrium action, both firm types have the identical set of beliefs under which the deviation is profitable. In this case, the D1 criterion imposes no restriction.

Second, consider an off-equilibrium action with $w' = 1$ and $P'(S,U) \in (1, 1+\phi' d]$ for uninformed quality-sensitive consumers; for other types of consumers, the firm sets personalized prices at their WTP levels. Then, the low-quality firm is willing to deviate when $1-\beta+\beta\alpha+\beta(1-\alpha)P'(S,U) - k > 1 (\Leftrightarrow P'(S,U) > P_l'(S,U) = \frac{1+k-(1-\beta+\beta\alpha)}{\beta(1-\alpha)})$, while the high-quality firm is willing to deviate when $1-\beta+\beta\alpha(1+d)+\beta(1-\alpha)P'(S,U) - k > \pi_h^{sep1*} (\Leftrightarrow P'(S,U) > P_h'(S,U) = \frac{\pi_h^{sep1*}-\beta\alpha d+k-(1-\beta+\beta\alpha)}{\beta(1-\alpha)})$, where $\pi_h^{sep1*} = \min\{\frac{1}{1-\alpha}, \beta(1+d)\}$.

If $P_h'(S,U) < P_l'(S,U)$, then for $P_h'(S,U) < P'(S,U) < P_l'(S,U)$, only the high-quality firm can benefit from the deviation; in this case, the out-of-equilibrium belief should be $\phi' = 1$. Given $\phi' = 1$, the high-quality firm will deviate, so the high-type-best PBE fails the D1 criterion. Note that $P_h'(S,U) < 1+d$ is always satisfied since $k < 1-\beta$ and $\pi_h^{sep1*} \leq \beta(1+d)$, so such a deviation exists.

By contrast, if $P_h'(S,U) > P_l'(S,U) (\Leftrightarrow \frac{1}{1+(1-\alpha)d} < \beta < \frac{1}{(1-\alpha)d})$, then whenever the high-quality firm can make a profitable deviation under a belief, the low-quality firm can do so too; in this case, it is easy to see that the high-type-best PBE survives the D1 criterion. Note that $P_l'(S,U) < P_h'(S,U) < 1+d$ and $P_l'(S,U) < 1+d \Leftrightarrow \beta > \frac{k}{(1-\alpha)d}$. Putting together, the high-type-best PBE survives the D1 criterion when $\max\{\frac{k}{(1-\alpha)d}, \frac{1}{1+(1-\alpha)d}\} < \beta < \frac{1}{(1-\alpha)d}$.

Refine all PBEs in Case 2 under the D1 criterion: In Case 2, a $pool2$-type PBE has $w_h^{pool2} = w_l^{pool2} = 0$ and $p_h^{pool2} = p_l^{pool2} = p^{pool2}$. Recall that we identified all such PBEs based on $IC_l^{pool2}: 1 \leq (1-\alpha)\beta p^{pool2}$ and $IC_h^{pool2}: \max\{\beta\alpha(1+d) + (1-\beta+\beta(1-\alpha)) - k, 1\} \leq \beta p^{pool2}$. The firm's equilibrium profit is $\pi_h^{pool2} = \beta p^{pool2}$ and $\pi_l^{pool2} = (1-\alpha)\beta p^{pool2}$. In particular, the high-type-best $pool2$-type PBE satisfies $\sigma_h^{pool2*} = (w_h^{pool2*} = 0, p_h^{pool2*} = 1+\lambda d)$ and $\sigma_l^{pool2*} = (w_l^{pool2*} = 0, p_l^{pool2*} = 1+\lambda d)$.

Now consider a $pool2$-type PBE with $\max\{\frac{1+\beta\alpha d-k}{\beta}, \frac{1}{(1-\alpha)\beta}\} \leq p^{pool2} \leq 1+\lambda d$. First, we consider an off-equilibrium action with $w' = 0$ and $p' > p^{pool2}$. For any belief ϕ' such that $p' \leq 1+\phi' d$, the low-quality firm is willing to deviate when $(1-\alpha)\beta p' > (1-\alpha)\beta p^{pool2} (\Leftrightarrow p' > p^{pool2})$, while the high-quality firm is willing to deviate when $\beta p' > \beta p^{pool2} (\Leftrightarrow p' > p^{pool2})$. Thus, for such an off-equilibrium action, both firm types have the identical set of beliefs under which the deviation is profitable. In this case, the D1 criterion imposes no restriction.

Second, we consider an off-equilibrium action with $w' = 1$ and $P'(S,U) \in (1, 1+\phi' d]$ for uninformed quality-sensitive consumers; for other types of consumers, the firm sets personalized prices at their WTP levels. Then, the low-quality firm is willing to deviate when $1-\beta+\beta\alpha+\beta(1-\alpha)P'(S,U) - k > (1-\alpha)\beta p^{pool2} (\Leftrightarrow P'(S,U) > P'_l(S,U) = \frac{(1-\alpha)\beta p^{pool2}+k-(1-\beta+\beta\alpha)}{\beta(1-\alpha)})$, while the high-quality firm is willing to deviate when $1-\beta+\beta\alpha(1+d)+\beta(1-\alpha)P'(S,U) - k > \beta p^{pool2} (\Leftrightarrow P'(S,U) > P'_h(S,U) = \frac{\beta p^{pool2}-\beta\alpha d+k-(1-\beta+\beta\alpha)}{\beta(1-\alpha)})$.

If $P'_h(S,U) > P'_l(S,U) (\Leftrightarrow p^{pool2} > d)$, then whenever the high-quality firm can make a profitable deviation under a belief, the low-quality firm can do so too; in this case, the PBE survives the D1 criterion. Similar to the above analysis for Case 1, we can eliminate the opposite case when $P'_h(S,U) < P'_l(S,U) (\Leftrightarrow p^{pool2} < d)$.

Thus, those $pool2$-type PBEs with $d < p^{pool2} \le 1+\lambda d$ can survive the D1 criterion. When $\lambda \le (d-1)/d$, none of the $pool2$-type PBEs we identified in the main analysis survives the D1 criterion.

<u>*Refine all PBEs in Case 3 under the D1 criterion:*</u> In Case 3, an $sep3$-type PBE has $w_h^{sep3} = 1$ and $w_l^{sep3} = 0$. Recall that we identified all $sep3$-type PBEs based on $IC_l^{sep3} : (1-\beta+\beta\alpha)+\beta(1-\alpha)P_h^{sep3}(S,U) - k \le 1$ and $IC_h^{sep3} : 1 \le \beta\alpha(1+d)+\beta(1-\alpha)P_h^{sep3}(S,U)+(1-\beta)-k$. The firm's equilibrium profit is $\pi_h^{sep3} = \beta\alpha(1+d)+\beta(1-\alpha)P_h^{sep3}(S,U)+(1-\beta)-k$ and $\pi_l^{sep3} = 1$. In particular, the high-type-best $sep3$-type PBE satisfies $\sigma_l^{sep3*} = (w_l^{sep3*} = 0, p_l^{sep3*} = 1)$ and $\sigma_h^{sep3*} = (w_h^{sep3*} = 1, P_h^{sep3*}(S,I) = 1+d, P_h^{sep3*}(S,U) = \min\{1+\frac{k}{\beta(1-\alpha)}, 1+d\}, P_h^{sep3*}(N,I) = 1, P_h^{sep3*}(N,U) = 1)$.

First, we show that all $sep3$-type PBEs except the high-type-best $sep3$-type PBE fail the D1 criterion. Suppose there exists an $sep3$-type PBE with $P_h^{sep3}(S,U) < P_h^{sep3*}(S,U) = \min\{1+\frac{k}{\beta(1-\alpha)}, 1+d\}$. Consider an off-equilibrium action with $w' = 1$ and $P'(S,U) = P_h^{sep3}(S,U)+\epsilon$, where $\epsilon > 0$ is sufficiently small; for other types of consumers, the firm sets personalized prices at their WTP levels. Then, under any belief ϕ', the low-quality firm is not willing to deviate because $1-\beta+\beta\alpha+\beta(1-\alpha)P'(S,U) - k \le 1$ (which holds since $P_h^{sep3*}(S,U)$ satisfies IC_l^{sep3}). However, the high-quality firm can profitably deviate with $\phi' = 1$. In other words, the high-quality firm benefits from a larger set of out-of-equilibrium beliefs than the low-quality firm. Thus, any $sep3$-type PBE with $P_h^{sep3}(S,U) < P_h^{sep3*}(S,U)$ fails the D1 criterion.

Next, we check whether the high-type-best $sep3$-type PBE survives the D1 criterion. In the case where $P_h^{sep3*}(S,U) = 1+d \le 1+\frac{k}{\beta(1-\alpha)}$, it is easy to see that the high-quality firm will never deviate because all personalized prices hit the WTP levels. Therefore, when $\beta \le \frac{k}{(1-\alpha)d}$, the high-type-best PBE has $P_h^{sep3*}(S,U) = 1+d$ and survives the D1 criterion. We proceed to examine $\beta > \frac{k}{(1-\alpha)d}$, in which case the high-type-best PBE has $P_h^{sep3*}(S,U) = 1+\frac{k}{\beta(1-\alpha)} < 1+d$.

First, we consider an off-equilibrium action with $w' = 1$ and $P'(S,U) > P_h^{sep3*}(S,U)$; for other types of consumers, the firm sets personalized prices at their WTP levels. Then, for $P'(S,U) \le 1 + \phi' d$, the low-quality firm is willing to deviate when $1 - \beta + \beta\alpha + \beta(1-\alpha)P'(S,U) - k > 1 = 1 - \beta + \beta\alpha + \beta(1-\alpha)P_h^{sep3*}(S,U) - k (\Leftrightarrow P'(S,U) > P_h^{sep3*}(S,U))$, while the high-quality firm is willing to deviate when $1 - \beta + \beta\alpha(1+d) + \beta(1-\alpha)P'(S,U) - k > 1 - \beta + \beta\alpha(1+d) + \beta(1-\alpha)P_h^{sep3*}(S,U) - k (\Leftrightarrow P'(S,U) > P_h^{sep3*}(S,U))$. Therefore, for such an off-equilibrium action, both firm types have the identical set of beliefs under which the deviation is profitable. In this case, the D1 criterion imposes no restriction.

Second, we consider an off-equilibrium action with $w' = 0$ and $p' \in (1, 1 + \phi' d]$. Then, the low-quality firm is willing to deviate when $\beta(1-\alpha)p' > 1 (\Leftrightarrow p' > p_l' = \frac{1}{\beta(1-\alpha)})$, while the high-quality firm is willing to deviate when $\beta p' > 1 + \beta\alpha d (\Leftrightarrow p' > p_h' = \frac{1+\beta\alpha d}{\beta})$.

If $p_h' < p_l' (\Leftrightarrow \beta < \frac{1}{(1-\alpha)d})$, then for $p_h' < p' < p_l'$, only the high-quality firm can benefit from the deviation; in this case, the out-of-equilibrium belief should be $\phi' = 1$. Given $\phi' = 1$, the high-quality firm will deviate, so the high-type-best PBE fails the D1 criterion. Note that only when $p_h' < 1 + d (\Leftrightarrow \beta > \frac{1}{1+(1-\alpha)d})$ can such a deviation exist. Conversely, when $\beta < \frac{1}{1+(1-\alpha)d}$, neither firm type will deviate, in which case the high-type-best PBE survives the D1 criterion.

If $p_h' > p_l' (\Leftrightarrow \beta > \frac{1}{(1-\alpha)d})$, then whenever the high-quality firm can make a profitable deviation under a belief, the low-quality firm can do so too; in this case, the high-type-best PBE with $P_h^{sep3*}(S,U) = 1 + \frac{k}{\beta(1-\alpha)}$ survives the D1 criterion.

Putting together, when $\max\{\frac{k}{(1-\alpha)d}, \frac{1}{1+(1-\alpha)d}\} < \beta < \frac{1}{(1-\alpha)d}$, none of the $sep3$-type PBEs we identified in the main analysis survives the D1 criterion.

Refine all PBEs in Case 4 under the D1 criterion: In Case 4, an $sep4$-type PBE has $w_h^{sep4} = w_l^{sep4} = 1$ and particularly $P_h^{sep4}(S,U) = P_l^{sep4}(S,U) = P^{sep4}(S,U)$. Recall that we identified all such PBEs based on $IC_l^{sep4}: 1 \le (1 - \beta + \beta\alpha) + \beta(1-\alpha)P_l^{sep4}(S,U) - k$ and $IC_h^{sep4}: 1 \le \beta\alpha(1+d) + \beta(1-\alpha)P_h^{sep4}(S,U) + (1-\beta) - k$. The firm's equilibrium profit is $\pi_h^{sep4} = \beta\alpha(1+d) + \beta(1-\alpha)P^{sep4}(S,U) + (1-\beta) - k$ and $\pi_l^{sep4} = (1 - \beta + \beta\alpha) + \beta(1-\alpha)P^{sep4}(S,U) - k$. In particular, the high-type-best PBE satisfies $\sigma_h^{sep4*} = (w_h^{sep4*} = 1, P_h^{sep4*}(S,I) = 1 + d, P_h^{sep4*}(N,I) = 1, P_h^{sep4*}(S,U) = 1 + \lambda d, P_h^{sep4*}(N,U) = 1)$ and $\sigma_l^{sep4*} = (w_l^{sep4*} = 1, P_l^{sep4*}(S,I) = 1, P_l^{sep4*}(N,I) = 1, P_l^{sep4*}(S,U) = 1 + \lambda d, P_l^{sep4*}(N,U) = 1)$.

Now consider an $sep4$-type PBE with $1 + \frac{k}{\beta(1-\alpha)} \le P^{sep4}(S,U) \le 1 + \lambda d$. First, we consider an off-equilibrium action with $w' = 1$ and $P'(S,U) > P^{sep4}(S,U)$; for other types of consumers, the firm sets personalized prices at their WTP levels. For any belief ϕ' such that $P'(S,U) \le 1 + \phi' d$, both the low-quality firm and the high-quality firm are willing to deviate because each firm's profit can increase by the same amount. Therefore, for such an off-equilibrium action, both firm types have the identical set of beliefs under which the deviation is profitable. In this case, the D1 criterion imposes no restriction.

Second, we consider an off-equilibrium action with $w' = 0$ and $p' \in (1, 1 + \phi' d]$. In this case, the low-quality firm is willing to deviate when $\beta(1-\alpha)p' > 1 - \beta + \beta\alpha + \beta(1-\alpha)P^{sep4}(S,U) - k \Leftrightarrow p' > p_l' =$

$\frac{1-\beta+\beta\alpha+\beta(1-\alpha)P^{sep4}(S,U)-k}{\beta(1-\alpha)}$, while the high-quality firm is willing to deviate when $\beta p' > 1-\beta+\beta\alpha(1+d)+\beta(1-\alpha)P^{sep4}(S,U)-k \Leftrightarrow p' > p'_h = \frac{1-\beta+\beta\alpha(1+d)+\beta(1-\alpha)P^{sep4}(S,U)-k}{\beta}$.

If $p'_h < p'_l (\Leftrightarrow P^{sep4}(S,U) > P_1 = 1+d-\frac{1-k}{\beta(1-\alpha)})$, then for $p'_h < p' < p'_l$, only the high-quality firm can benefit from the deviation; in this case, the out-of-equilibrium belief should be $\phi' = 1$. Given $\phi' = 1$, the high-quality firm will deviate, so the PBE fails the D1 criterion. Note that only when $p'_h < 1+d (\Leftrightarrow P^{sep4}(S,U) < P_2 = 1+d-\frac{1-k}{\beta(1-\alpha)}+\frac{1}{1-\alpha})$ can such a deviation exist. Conversely, when $P^{sep4}(S,U) > P_2$, neither firm type will deviate, in which case the PBE survives the D1 criterion.

By contrast, if $p'_h > p'_l (\Leftrightarrow P^{sep4}(S,U) < P_1)$, then whenever the high-quality firm can make a profitable deviation under a belief, the low-quality firm can do so too; in this case, the PBE survives the D1 criterion.

Given $1+\frac{k}{\beta(1-\alpha)} \leq P^{sep4}(S,U) \leq 1+\lambda d$, one can show that when $\beta < \frac{1}{(1-\alpha)d} (\Leftrightarrow P_1 < 1+\frac{k}{\beta(1-\alpha)})$ and $\beta > \frac{1-k}{1+(1-\alpha)(1-\lambda)d} (\Leftrightarrow P_2 > 1+\lambda d)$, none of the $sep4$-type PBEs we identified in the main analysis survives the D1 criterion.

<u>*Summary:*</u> Recall from the main analysis that when β, d, and α are moderate, the SUE refinement uniquely selects the high-type-best $sep1$-type PBE: $\sigma_h^{sep1*} = (w_h^{sep1*} = 0, p_h^{sep1*} = \min\{\frac{1}{(1-\alpha)\beta}, 1+d\})$ and $\sigma_l^{sep1*} = (w_l^{sep1*} = 0, p_l^{sep1*} = 1)$, a *no-adoption* equilibrium.

We find that $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ can also *uniquely* survive the D1 criterion when β, d, and α are moderate. This result demonstrates the robustness of our key insight that the high-quality firm may refrain from adopting PP to signal its high quality. Specifically, we have shown above that $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ survives the D1 criterion when $\max\{\frac{k}{(1-\alpha)d}, \frac{1}{1+(1-\alpha)d}\} < \beta < \frac{1}{(1-\alpha)d}$; in the meantime, $sep3$-type PBE is ruled out by the D1 criterion. In addition, we consider $\beta < \frac{1}{(1-\alpha)(1+\lambda d)}$ (under which $pool2$-type PBE does not exist) and $\beta < \frac{k}{(1-\alpha)\lambda d}$ (under which $sep4$-type PBE does not exist). Together, these conditions imply that $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ is the *unique* equilibrium that survives the D1 criterion. Combining them, the no-adoption equilibrium uniquely survives the D1 criterion when $\max\left\{\frac{k}{(1-\alpha)d}, \frac{1}{1+(1-\alpha)d}\right\} < \beta < \min\left\{\frac{1}{(1-\alpha)d}, \frac{1}{(1-\alpha)(1+\lambda d)}, \frac{k}{(1-\alpha)\lambda d}\right\}$, which can be equivalently expressed in terms of α as $\max\left\{1-\frac{1}{\beta d}, 1-\frac{1}{\beta(1+\lambda d)}, 1-\frac{k}{\beta\lambda d}\right\} < \alpha < \min\left\{1-\frac{k}{\beta d}, 1-\frac{1-\beta}{\beta d}\right\}$, or in terms of d as $\max\left\{\frac{1-\beta}{\beta(1-\alpha)}, \frac{k}{\beta(1-\alpha)}\right\} < d < \min\left\{\frac{1}{\beta(1-\alpha)}, \frac{1}{\lambda}\left(\frac{1}{\beta(1-\alpha)}-1\right), \frac{k}{\beta\lambda(1-\alpha)}\right\}$. Note that these conditions represent moderate ranges of β, d, or α, hence Lemma 2. These conditions can be simultaneously satisfied; for example, when $\lambda < k$ and $\lambda < (d-1)/d$.

We also find that the D1 criterion does not generally ensure a unique equilibrium in our setting. As an illustrative example, both the high-type-best $sep1$-type PBE and the high-type-best $pool2$-type PBE can survive the D1 criterion when $\frac{1}{(1-\alpha)(1+\lambda d)} < \beta < \frac{1}{(1-\alpha)d}$ (which is nonempty if $\lambda > (d-1)/d$.) This non-uniqueness under D1 further implies that another commonly used but weaker refinement, the intuitive criterion, also fails to ensure the equilibrium uniqueness in our model.

Online Appendix B: Proofs of results for the imperfect-targeting model

OB.1. Equilibrium derivation for the imperfect-targeting model

We derive the equilibrium of the signaling game for the extension model where the firm knows each consumer's valuation θ but not informedness τ.

Table B1. Potential equilibrium cases under imperfect targeting

	Separating ($\sigma_h \neq \sigma_l$)	Pooling ($\sigma_h = \sigma_l$)
$w_h = w_l = 0$	Case 1: PBE exists, denoted as $sep1$-type	Case 2: PBE exists, denoted as $pool2$-type
$w_h = 1, w_l = 0$	Case 3: PBE exists, denoted as $sep3$-type	N/A
$w_h = w_l = 1$	Case 4: No PBE exists	Case 5: PBE exists, denoted as $pool5$-type
$w_h = 0, w_l = 1$	Case 6: No PBE exists	N/A

Table B1 summarizes the potential equilibrium cases. We can preliminarily rule out Case 6. This is because if the low-quality firm chooses $w_l = 1$ in a separating equilibrium where $w_h = 0$, its profit will be $1-k$. However, it can profitably deviate to $\sigma' = (w = 0, p = 1)$, which yields a higher profit of 1. In the rest of the proof, we will examine Cases 1 (separating), 2 (pooling), 3 (separating), 4 (separating), and 5 (pooling). We denote the PBE in the five cases respectively as $sep1$-type, $pool2$-type, $sep3$-type, $sep4$-type, and $pool5$-type.

Similar to the analysis for the main model, we solve for the SUE, and we assume the pessimistic beliefs: $\phi(\gamma_i(\sigma)) = 0$ if $\gamma_i(\sigma) \notin \{\gamma_i(\sigma_h), \gamma_i(\sigma_l)\}$. The rest of the proof takes six steps: (i) In Case 1, we find the high-type-best $sep1$-type PBE $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$, which yields the highest profit for the high-quality firm among all $sep1$-type PBEs. (ii) In Case 2, we find the high-type-best $pool2$-type PBE $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$, which yields the highest profit for the high-quality firm among all $pool2$-type PBEs. (iii) In Case 3, we find the high-type-best $sep3$-type PBE $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$, which yields the highest profit for the high-quality firm among all $sep3$-type PBEs. (iv) In Case 4, we prove that the $sep4$-type PBE does not exist. (v) In Case 5, we find the high-type-best $pool5$-type PBE $(\sigma_h^{pool5*}, \sigma_l^{pool5*})$, which yields the highest profit for the high-quality firm among all $pool5$-type PBEs. (vi) Lastly, we compare $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$, $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$, $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$, and $(\sigma_h^{pool5*}, \sigma_l^{pool5*})$ to derive the SUE that yields the highest profit for the high-quality firm. To proceed, it is without loss of generality to assume that the firm's prices lie on the range of $[1, 1+d]$ throughout the analysis. The reason is that since consumers' willingness to pay lies between 1 and $1+d$, setting (personalized or uniform) prices strictly below 1 or strictly above $1+d$ is strictly sub-optimal.

Finding the high-type-best PBE for each case

Step (i): Find the high-type-best equilibrium for Case 1 $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$

Let $(\sigma_h^{sep1}, \sigma_l^{sep1}, \phi)$ be an $sep1$-type PBE in Case 1, where $w_h^{sep1} = w_l^{sep1} = 0$ and $\sigma_h^{sep1} \neq \sigma_l^{sep1}$. We find $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ in two steps. In step one, we derive the IC (incentive-compatible) constraints for both firm types, which must be satisfied in equilibrium. In step two, we find $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$—the $sep1$-type PBE that yields the highest profit for the high-quality firm and satisfies the IC constraints.

Step 1. We start with the low-quality firm. In an $sep1$-type equilibrium, it must be that $w_l^{sep1} = 0$ and $p_l^{sep1} = 1$, which leads to $\pi_l^{sep1} = 1$. It follows that $p_h^{sep1} \in (1, 1+d]$ must be true in the separating equilibrium.

We derive the low-quality firm's IC constraint. We check all possible deviations by discussing the following three cases. First, the low-quality firm will not deviate to a σ' with $w(\sigma') = 1$, because it at best leads to a deviation profit of $\pi_l(\sigma') = 1 - k$, lower than π_l^{sep1}. Second, the low-quality firm will not deviate to a σ' with $w(\sigma') = 0$ and $p(\sigma') \neq p_h^{sep1}$. This is because all uninformed consumers will believe that the firm has low quality with probability one and hence its deviation profit must be lower than π_l^{sep1}. Third, we check the low-quality firm's deviation to $\sigma' = \sigma_h^{sep1}$, i.e., $w(\sigma') = 0$ and $p(\sigma') = p_h^{sep1}$. Under this deviation, the low-quality firm's deviation profit will be $\pi_l(\sigma') = (1-\alpha)\beta p_h^{sep1}$. That is, the low-quality firm will only sell to uninformed quality-sensitive consumers by mimicking the high-quality firm's action σ_h^{sep1}. In this case, the low-quality firm will not deviate if and only if $(1-\alpha)\beta p_h^{sep1} \leq 1$. Summarizing the above cases, the low-quality firm's IC constraint is given by

$$(IC_l^{sep1}) : (1-\alpha)\beta p_h^{sep1} \leq 1.$$

Next, we consider the high-quality firm. In equilibrium, all consumers believe that the product has high quality with probability one. Thus, the high-quality firm's equilibrium profit is $\pi_h^{sep1} = \beta p_h^{sep1}$, where $p_h^{sep1} \in (1, 1+d]$. We check all possible deviations by discussing the following cases.

If $p_h^{sep1} = 1+d$, the high-quality firm will not deviate to a σ' with $w(\sigma') = 0$, because the deviation profit must be lower than $\pi_h^{sep1} = \beta(1+d)$. Next, we consider $p_h^{sep1} \in (1, 1+d)$ and a deviation σ' with $w(\sigma') = 0$. In this case, we show that the high-quality firm will not deviate to a σ' with $w(\sigma') = 0$ and $p(\sigma') \in (1, p_h^{sep1})$, because the deviation profit $\pi_h(\sigma') = \beta\alpha p(\sigma')$ must be lower than $\pi_h^{sep1} = \beta p_h^{sep1}$. However, we need to check a deviation σ' with $w(\sigma') = 0$ and $p(\sigma') = 1$, where the deviation profit is $\pi_h(\sigma') = 1$. Moreover, we need to check a deviation σ' with $w(\sigma') = 0$ and $p(\sigma') = 1+d$, which is the optimal deviation among those deviations with $w(\sigma') = 0$ and $p(\sigma') > p_h^{sep1}$. The deviation profit is $\pi_h(\sigma') = \beta\alpha(1+d)$.

In addition, we consider a deviation σ' with $w(\sigma') = 1$. Under this deviation, for quality-insensitive consumers, the high-quality firm will optimally set $P(\sigma', N) = 1$; for quality-sensitive consumers, the high-quality firm will set either $P(\sigma', S) = 1+d$ (capturing only informed quality-sensitive consumers) or $P(\sigma', S) = 1$ (capturing all quality-sensitive consumers). Thus, the high-quality firm's deviation profit is $\pi_h(\sigma') = 1 - \beta + \beta \max\{\alpha(1+d), 1\} - k$. In the latter case, the deviation profit $1-k$ is smaller than one.

Summarizing the above cases, the high-quality firm's IC constraint is given by

$$(IC_h^{sep1}) : \max\{1, 1-\beta+\beta\alpha(1+d)-k\} \leq \beta p_h^{sep1}.$$

Step 2. We find $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ by accounting for both (IC_l^{sep1}) and (IC_h^{sep1}). We divide the discussion into four parameter regions. Recall that in Case 1, $p_h^{sep1} \in (1, 1+d]$ must be true.

In region 1, $\beta \leq \frac{1}{(1-\alpha)(1+d)}$ and $k < 1-\beta+\beta\alpha(1+d)-\beta(1+d)$. In this region, (IC_h^{sep1}) is violated. So, no $sep1$-type PBE exists.

In region 2, $\beta \leq \frac{1}{(1-\alpha)(1+d)}$ and $k \geq 1-\beta+\beta\alpha(1+d)-\beta(1+d)$. In this region, one can show that the high-type-best $sep1$-type PBE satisfies $\sigma_h^{sep1*} = (w_h^{sep1*} = 0, p_h^{sep1*} = 1+d)$ and $\sigma_l^{sep1*} = (w_l^{sep1*} = 0, p_l^{sep1*} = 1)$. First, (IC_l^{sep1}) is satisfied $\forall p_h^{sep1} \in (1, 1+d]$. Second, among all the p_h^{sep1} that satisfies (IC_h^{sep1}), $p_h^{sep1} = 1+d$ yields the highest profit for the high-quality firm.

In region 3, $\beta > \frac{1}{(1-\alpha)(1+d)}$ and $k < 1-\beta+\beta\alpha(1+d)-\frac{1}{1-\alpha}$. In this region, $p_h^{sep1} = 1+d$ does not satisfy (IC_l^{sep1}). Instead, (IC_l^{sep1}) is satisfied for $p_h^{sep1} \in (1, \frac{1}{(1-\alpha)\beta}]$. However, for $p_h^{sep1} \in (1, \frac{1}{(1-\alpha)\beta}]$, (IC_h^{sep1}) is violated. So, no $sep1$-type PBE exists.

In region 4, $\beta > \frac{1}{(1-\alpha)(1+d)}$ and $k \geq 1-\beta+\beta\alpha(1+d)-\frac{1}{1-\alpha}$. In this region, we show that the high-type-best $sep1$-type PBE satisfies $\sigma_h^{sep1*} = (w_h^{sep1*} = 0, p_h^{sep1*} = \frac{1}{(1-\alpha)\beta})$ and $\sigma_l^{sep1*} = (w_l^{sep1*} = 0, p_l^{sep1*} = 1)$. This is because among possible p_h^{sep1}, $p_h^{sep1} = \frac{1}{(1-\alpha)\beta}$ yields the highest profit for the high-quality firm; moreover, both (IC_l^{sep1}) and (IC_h^{sep1}) are satisfied under this PBE.

To sum up, when $k \geq 1-\beta+\beta\alpha(1+d)-\beta\min\{\frac{1}{(1-\alpha)\beta}, 1+d\}$, the $sep1$-type PBE exists. Furthermore, the high-type-best $sep1$-type equilibrium satisfies $\sigma_h^{sep1*} = (w_h^{sep1*} = 0, p_h^{sep1*} = \min\{\frac{1}{(1-\alpha)\beta}, 1+d\})$ and $\sigma_l^{sep1*} = (w_l^{sep1*} = 0, p_l^{sep1*} = 1)$. In this high-type-best equilibrium, $\pi_h^{sep1*} = \beta\min\{\frac{1}{(1-\alpha)\beta}, 1+d\}$ and $\pi_l^{sep1*} = 1$.

Step (ii): Find the high-type-best equilibrium for Case 2 $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$

Let $(\sigma_h^{pool2}, \sigma_l^{pool2}, \phi)$ be a $pool2$-type PBE in Case 2, where $w_h^{pool2} = w_l^{pool2} = 0$ and $\sigma_h^{pool2} = \sigma_l^{pool2}$, hence $p_h^{pool2} = p_l^{pool2}$. The same pooling price is denoted as p^{pool2}. Below, we focus on the case where $p^{pool2} > 1$ and find $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$ in two steps.[4] In step one, we derive the IC constraints for both firm types, which must be satisfied in equilibrium. In step two, we find $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$—the $pool2$-type PBE that yields the highest profit for the high-quality firm and satisfies the IC constraints.

Step 1. In a $pool2$-type equilibrium, uninformed consumers' quality belief is $\phi(\gamma(\sigma)) = \lambda$ if $\sigma = \sigma_j^{pool2}$, and $\phi(\gamma(\sigma)) = 0$ if $\sigma \neq \sigma_j^{pool2}$. Therefore, in equilibrium, $p^{pool2} \leq 1+\lambda d$.

[4] In the special $pool2$-type equilibrium where $w_h^{pool2} = w_l^{pool2} = 0$ and $p_h^{pool2} = p_l^{pool2} = 1$, we have $\pi_h^{pool2} = \pi_l^{pool2} = 1$. To sustain this equilibrium, we need $1 \geq 1-\beta+\beta\alpha(1+d)-k$ such that the high-quality firm will not deviate to a σ' with $w(\sigma') = 1$, $P(\sigma', S) = 1+d$, and $P(\sigma', N) = 1$. So, this equilibrium exists when $k \geq \beta\alpha(1+d)-\beta$. Then, it is easy to show that whenever this $pool2$-type equilibrium exists, the high-type-best $sep1$-type equilibrium also exists and dominates this $pool2$-type equilibrium. Therefore, we consider $p^{pool2} > 1$.

We first derive the low-quality firm's IC constraint. Since $p^{pool2} > 1$, the low-quality firm's equilibrium profit is $\pi_l^{pool2} = (1-\alpha)\beta p^{pool2}$ by capturing uninformed quality-sensitive consumers. One can readily show that among all possible deviations for the low-quality firm, the best deviation is to charge a uniform price of one and receive a profit of $\pi_l(\sigma') = 1$. Thus, the low-quality firm's IC constraint is given by

$$(IC_l^{pool2}) : 1 \leq (1-\alpha)\beta p^{pool2}.$$

Next, we consider the high-quality firm. Since $p^{pool2} > 1$, the high-quality firm's equilibrium profit is $\pi_h^{pool2} = \beta p^{pool2}$ by capturing quality-sensitive consumers. We check all possible deviations by discussing the following cases. First, if the high-quality firm deviates to a σ' with $w(\sigma') = 0$ and $p(\sigma') > p^{pool2}$, then the optimal deviation should be $p(\sigma') = 1+d$, leading to a deviation profit of $\pi_h(\sigma') = \beta\alpha(1+d)$ by capturing only informed quality-sensitive consumers. Second, the high-quality firm will not deviate to any σ' with $w(\sigma') = 0$ and $p(\sigma') \in (1, p^{pool2})$, because the deviation profit $\pi_h(\sigma') = \beta\alpha p(\sigma')$ must be lower than the equilibrium profit π_h^{pool2}. Third, if the high-quality firm deviates to a σ' with $w(\sigma') = 0$ and $p(\sigma') = 1$, then the deviation profit is $\pi_h(\sigma') = 1$. Lastly, if the high-quality firm deviates to a σ' with $w(\sigma') = 1$, for quality-insensitive consumers, the high-quality firm will optimally set $P(\sigma', N) = 1$; for quality-sensitive consumers, the high-quality firm will set either $P(\sigma', S) = 1+d$ (capturing only informed quality-sensitive consumers) or $P(\sigma', S) = 1$ (capturing all quality-sensitive consumers), leading to a deviation profit of $\pi_h(\sigma') = 1-\beta+\beta\max\{\alpha(1+d), 1\} - k$. Summarizing the above cases, the high-quality firm's IC constraint is given by

$$(IC_h^{pool2}) : \max\{1, 1-\beta+\beta\alpha(1+d)-k\} \leq \beta p^{pool2}.$$

Step 2. We find $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$ by accounting for both (IC_l^{pool2}) and (IC_h^{pool2}). Putting together, we have $\max\{\frac{1-\beta+\beta\alpha(1+d)-k}{\beta}, \frac{1}{(1-\alpha)\beta}\} \leq p^{pool2} \leq 1+\lambda d$. So, the $pool2$-type equilibrium exists when $\beta \geq \frac{1}{(1-\alpha)(1+\lambda d)}$ and $k \geq 1-\beta+\beta\alpha(1+d)-\beta(1+\lambda d)$. Among all possible PBEs, the one with $p^{pool2} = 1+\lambda d$ yields the highest profit for the high-quality firm. So, the high-type-best $pool2$-type equilibrium satisfies $\sigma_h^{pool2*} = (w_h^{pool2*} = 0, p_h^{pool2*} = 1+\lambda d)$ and $\sigma_l^{pool2*} = (w_l^{pool2*} = 0, p_l^{pool2*} = 1+\lambda d)$. In this high-type-best equilibrium, $\pi_h^{pool2*} = \beta(1+\lambda d)$ and $\pi_l^{pool2*} = (1-\alpha)\beta(1+\lambda d)$.

Step (iii): Find the high-type-best equilibrium for Case 3 $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$

Let $(\sigma_h^{sep3}, \sigma_l^{sep3}, \phi)$ be an $sep3$-type PBE in Case 3, where $w_h^{sep3} = 1$ and $w_l^{sep3} = 0$, hence $\sigma_h^{sep3} \neq \sigma_l^{sep3}$. We find $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$ in two steps. In step one, we derive the IC constraints for both firm types, which must be satisfied in equilibrium. In step two, we find $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$—the $sep3$-type PBE that yields the highest profit for the high-quality firm and satisfies the IC constraints.

Step 1. In an $sep3$-type equilibrium, the high-quality firm must charge $P_h^{sep3}(N) = 1$ for quality-insensitive consumers. For quality-sensitive consumers, suppose that the high-quality firm charges $P_h^{sep3}(S)$, without knowing whether they are informed or not. It is easy to see that $P_h^{sep3}(S) > 1$, because otherwise the high-quality firm would profitably deviate to not adopting the PP technology. In an $sep3$-type equilibrium, it must be that $w_l^{sep3} = 0$ and $p_l^{sep3} = 1$, which leads to $\pi_l^{sep3} = 1$.

We first derive the low-quality firm's IC constraint. We check all possible deviations by discussing the following three cases. First, the low-quality firm will not deviate to a σ' with $w(\sigma') = 0$ and $p_l^{sep3} \neq 1$, because the deviation profit must be lower than 1. Second, the low-quality firm will not deviate to a σ' with $w(\sigma') = 1$ and $P(\sigma', S) \neq P_h^{sep3}(S)$ for quality-sensitive consumers. Since uninformed quality-sensitive consumers will believe that the product has low quality with probability one, its deviation profit must be lower than 1 because of the adoption cost k. Third, we consider a deviation σ' with $w(\sigma') = 1$ and $P(\sigma', S) = P_h^{sep3}(S)$ for quality-sensitive consumers; for quality-insensitive consumers, the firm will optimally set $P(\sigma', N) = 1$. The deviation profit is $\pi_l(\sigma') = 1 - \beta + \beta(1-\alpha)P_h^{sep3}(S) - k$. In this case, the low-quality firm will not deviate if and only if $1 - \beta + \beta(1-\alpha)P_h^{sep3}(S) - k \leq 1$. Summarizing the three cases, the low-quality firm's IC constraint is given by

$$(IC_l^{sep3}) : 1 - \beta + \beta(1-\alpha)P_h^{sep3}(S) - k \leq 1.$$

Next, we derive the high-quality firm's IC constraint. In an $sep3$-type equilibrium, the high-quality firm's profit is $\pi_h^{sep3} = 1 - \beta + \beta P_h^{sep3}(S) - k$, where $P_h^{sep3}(S) \leq 1 + d$. We check all possible deviations by discussing the following cases. First, the high-quality firm will not deviate to a σ' with $w(\sigma') = 1$ and $P(\sigma', S) < P_h^{sep3}(S)$. Since uninformed quality-sensitive consumers will believe that the product has low quality with probability one, the high-quality firm cannot earn higher profit. Second, if $P_h^{sep3}(S) < 1 + d$, we need to check a deviation σ' with $w(\sigma') = 1$ and $P(\sigma', S) = 1 + d$, which is the optimal deviation among those deviations with $w(\sigma') = 1$ and $P(\sigma', S) > P_h^{sep3}(S)$, leading to a deviation profit of $\pi_h(\sigma') = 1 - \beta + \beta\alpha(1+d) - k$. That is, instead of capturing all quality-sensitive consumers, the high-quality firm may choose to capture only informed quality-sensitive consumers. Third, we consider a deviation σ' with $w(\sigma') = 0$ and $p(\sigma') = 1$, where the deviation profit is $\pi_h(\sigma') = 1$. Fourth, we consider a deviation σ' with $w(\sigma') = 0$ and $p(\sigma') = 1 + d$, which is the optimal deviation among those deviations with $w(\sigma') = 0$ and $p(\sigma') > 1$. The deviation profit is $\pi_h(\sigma') = \beta\alpha(1+d)$. Summarizing the above cases, the high-quality firm's IC constraint is given by

$$(IC_h^{sep3}) : \max\{1, 1 - \beta + \beta\alpha(1+d) - k\} \leq 1 - \beta + \beta P_h^{sep3}(S) - k.$$

Step 2. We find $(\sigma_l^{sep3*}, \sigma_h^{sep3*})$ by accounting for both (IC_l^{sep3}) and (IC_h^{sep3}). Putting the IC constraints together, we have $P_h^{sep3}(S) \in [\max\{\frac{\beta+k}{\beta}, \alpha(1+d)\}, \min\{\frac{\beta+k}{\beta(1-\alpha)}, 1+d\}]$. Note that equation (3) implies $k < \beta d$. One can show that when $k < \beta\alpha(1-\alpha)(1+d) - \beta$, the $sep3$-type PBE does not exist. Otherwise, when $k \geq \beta\alpha(1-\alpha)(1+d) - \beta$, among all possible PBEs, the one with $P_h^{sep3}(S) = \min\{\frac{\beta+k}{\beta(1-\alpha)}, 1+d\}$

yields the highest profit for the high-quality firm. Specifically, when $k \geq \beta(1-\alpha)(1+d)-\beta$, $P_h^{sep3}(S) = 1+d$; when $\beta\alpha(1-\alpha)(1+d)-\beta \leq k < \beta(1-\alpha)(1+d)-\beta$, $P_h^{sep3}(S) = \frac{\beta+k}{\beta(1-\alpha)}$. To sum up, the high-type-best $sep3$-type equilibrium satisfies $\sigma_l^{sep3*} = (w_l^{sep3*} = 0, p_l^{sep3*} = 1)$ and $\sigma_h^{sep3*} = (w_h^{sep3*} = 1, P_h^{sep3*}(S) = \min\{\frac{\beta+k}{\beta(1-\alpha)}, 1+d\}, P_h^{sep3*}(N) = 1)$. In this high-type-best equilibrium, we have $\pi_h^{sep3*} = 1-\beta+\beta\min\{\frac{\beta+k}{\beta(1-\alpha)}, 1+d\}-k$ and $\pi_l^{sep3*} = 1$.

Step (iv): No PBE exists in Case 4

Let $(\sigma_h^{sep4}, \sigma_l^{sep4}, \phi)$ be an $sep4$-type PBE in Case 4, where $w_h^{sep4} = w_l^{sep4} = 1$ and $\sigma_h^{sep4} \neq \sigma_l^{sep4}$. We prove that an $sep4$-type PBE does not exist by showing that $P_h^{sep4}(S) = P_l^{sep4}(S)$ and $P_h^{sep4}(N) = P_l^{sep4}(N)$ must hold in equilibrium. First, it is clear that $P_h^{sep4}(N) = P_l^{sep4}(N) = 1$. Second, we show $P_h^{sep4}(S) = P_l^{sep4}(S)$. Suppose not, then for uninformed quality-sensitive consumers, $\gamma(\sigma_h^{sep4}) \neq \gamma(\sigma_l^{sep4})$, which implies $\phi(\gamma(\sigma_l^{sep4})) = 0$, hence $P_l^{sep4}(S) \leq 1$. Since $P_l^{sep4}(S) \leq 1$, the low-quality firm would profitably deviate to not adopting the PP technology. Therefore, it must hold that $P_h^{sep4}(S) = P_l^{sep4}(S)$ and hence $\sigma_h^{sep4} = \sigma_l^{sep4}$.

Step (v): Find the high-type-best equilibrium for Case 5 $(\sigma_h^{pool5*}, \sigma_l^{pool5*})$

Let $(\sigma_h^{pool5}, \sigma_l^{pool5}, \phi)$ be a $pool5$-type PBE in Case 5, where $w_h^{pool5} = w_l^{pool5} = 1$ and $\sigma_h^{pool5} = \sigma_l^{pool5}$. Based on Step (iv), we know $\phi(\gamma(\sigma_h^{pool5})) = \phi(\gamma(\sigma_l^{pool5})) = \lambda$. This implies $P_h^{pool5}(S) = P_l^{pool5}(S) \leq 1+\lambda d$. Moreover, for quality-insensitive consumers, $P_h^{pool5}(N) = P_l^{pool5}(N) = 1$. We find $(\sigma_h^{pool5*}, \sigma_l^{pool5*})$ in two steps. In step one, we derive the IC constraints for both firm types, which must be satisfied in equilibrium. In step two, we find $(\sigma_h^{pool5*}, \sigma_l^{pool5*})$—the $pool5$-type PBE that yields the highest profit for the high-quality firm and satisfies the IC constraints.

Step 1. In equilibrium, the high-quality firm's profit is $\pi_h^{pool5} = 1-\beta+\beta P_h^{pool5}(S)-k$, while the low-quality firm's profit is $\pi_l^{pool5} = 1-\beta+\beta(1-\alpha)P_l^{pool5}(S)-k$. Next, we check all possible deviations for the high-quality and low-quality firm, respectively.

Consider the low-quality firm. First, we consider a deviation σ' with $w(\sigma') = 0$. The optimal deviation profit is $\pi_l(\sigma') = 1$. Then, the low-quality firm will not deviate if and only if $1 \leq \pi_l^{pool5}$. Second, we consider a deviation σ' with σ_l^{pool5} except $P(\sigma', S) \neq P_l^{pool5}(S)$. Since uninformed quality-sensitive consumers will believe that the product has low quality with probability one, the deviation profit is not greater than $1-k$. To sum up, the low-quality firm's IC constraint is given by

$$(IC_l^{pool5}) : 1 \leq 1-\beta+\beta(1-\alpha)P_l^{pool5}(S)-k.$$

Consider the high-quality firm. First, the high-quality firm will not deviate to a σ' with $w(\sigma') = 1$ and $P(\sigma', S) < P_h^{pool5}(S)$. Since uninformed quality-sensitive consumers will believe that the product has low quality with probability one, the high-quality firm cannot earn higher profit. Second, if $P_h^{pool5}(S) < 1+d$, we need to check a deviation σ' with $w(\sigma') = 1$ and $P(\sigma', S) = 1+d$, which is the optimal deviation among those deviations with $w(\sigma')=1$ and $P(\sigma',S) > P_h^{pool5}(S)$, leading to a deviation profit of $\pi_h(\sigma') = 1-\beta+\beta\alpha(1+d)-k$. That is, instead of capturing all quality-sensitive consumers, the high-quality firm may choose to capture only informed quality-sensitive consumers. Third, we consider a deviation σ' with $w(\sigma')=0$ and $p(\sigma')=1$, where the deviation profit is $\pi_h(\sigma')=1$. Fourth, we consider a deviation σ' with $w(\sigma')=0$ and $p(\sigma')=1+d$, which is the optimal deviation with $w(\sigma')=0$ and $p(\sigma')>1$, leading to a deviation profit of $\pi_h(\sigma')=\beta\alpha(1+d)$. Summarizing the above cases, the high-quality firm's IC constraint is given by

$$(IC_h^{pool5}) : \max\{1, 1-\beta+\beta\alpha(1+d)-k\} \le 1-\beta+\beta P_h^{pool5}(S)-k.$$

Step 2. Putting the IC constraints together, we have $\max\{\frac{\beta+k}{\beta(1-\alpha)}, \alpha(1+d)\} \le P_h^{pool5}(S) = P_l^{pool5}(S) \le 1+\lambda d$. Thus, the $pool5$-type PBE exists when $k \le \beta(1-\alpha)(1+\lambda d)-\beta = \beta(1-\alpha)\lambda d - \beta\alpha$ and $\alpha \le \frac{1+\lambda d}{1+d}$. Moreover, $P_h^{pool5}(S) = P_l^{pool5}(S) = 1+\lambda d$ yields the highest profit for the high-quality firm. Thus, the high-type-best $pool5$-type equilibrium satisfies $\sigma_h^{pool5*} = (w_h^{pool5*}=1, P_h^{pool5*}(S)=1+\lambda d, P_h^{pool5*}(N)=1)$ and $\sigma_l^{pool5*} = (w_l^{pool5*}=1, P_l^{pool5*}(S)=1+\lambda d, P_l^{pool5*}(N)=1)$. In this high-type-best equilibrium, we have $\pi_h^{pool5*} = 1-\beta+\beta(1+\lambda d)-k$ and $\pi_l^{pool5*} = 1-\beta+\beta(1-\alpha)(1+\lambda d)-k$.

Deriving the SUE across parameters

Step (vi): Derive the SUE

We compare $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$, $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$, $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$, and $(\sigma_h^{pool5*}, \sigma_l^{pool5*})$ to derive the SUE that yields the highest profit for the high-quality firm. We will show the results in Table B2 with respect to parameter β, which can also be translated into other parameter perspectives, as in the main model.

Let $k_1 \equiv \beta(1-\alpha)(1+\lambda d)-\beta$, $k_2 \equiv \beta\alpha(1-\alpha)(1+d)-\beta$, and $k_3 \equiv \beta(1-\alpha)(1+d)-\beta$. We divide the discussion into three parameter regions: $k \ge k_3$, $\max\{k_1,k_2\} < k < k_3$, and $k \le \max\{k_1,k_2\}$. In the following analysis, we do not explicitly repeat the assumptions in equation (3) (e.g., $\beta < 1-k$) unless we need a boundary for complete definitions of the thresholds.

Region 1: $k \ge k_3$. With respect to β, this region is equivalent to $\beta \le \hat{\beta}_1$, where

$$\hat{\beta}_1 \equiv \begin{cases} 1-k & \text{if } \alpha \ge \frac{d}{1+d}, \\ \frac{k}{(1-\alpha)(1+d)-1} & \text{if } \alpha < \frac{d}{1+d}. \end{cases}$$

In this region, the $pool5$-type PBE does not exist because $k \ge k_3 > k_1$. Moreover, $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ and $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$ must be dominated by $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$, because $\pi_h^{sep3*} = 1-\beta+\beta(1+d)-k$ is larger than $\pi_h^{sep1*} = \beta(1+d)$ and $\pi_h^{pool2*} = \beta(1+\lambda d)$. Therefore, in Region 1, the SUE is $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$.

Region 2: $\max\{k_1,k_2\} < k < k_3$. With respect to β, this region is equivalent to $\hat{\beta}_1 < \beta < \hat{\beta}_2$, where

$$\hat{\beta}_2 \equiv \begin{cases} \frac{k}{\alpha(1-\alpha)(1+d)-1} & \text{if } d > \frac{1-\alpha+\alpha^2}{\alpha-\alpha^2} \text{ and } \lambda < \frac{\alpha(1+d)-1}{d}, \\ \frac{k}{(1-\alpha)(1+\lambda d)-1} & \text{if } d > \frac{1-\alpha+\alpha^2}{\alpha-\alpha^2} \text{ and } \lambda \ge \frac{\alpha(1+d)-1}{d}, \\ 1-k & \text{if } d \le \frac{1-\alpha+\alpha^2}{\alpha-\alpha^2} \text{ and } \lambda \le \frac{\alpha}{(1-\alpha)d}, \\ \frac{k}{(1-\alpha)(1+\lambda d)-1} & \text{if } d \le \frac{1-\alpha+\alpha^2}{\alpha-\alpha^2} \text{ and } \lambda > \frac{\alpha}{(1-\alpha)d}. \end{cases}$$

In this region, the $pool5$-type PBE still does not exist. Moreover, $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$ must be dominated by $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$, because $\pi_h^{sep3*} = 1-\beta+\frac{\beta+k}{1-\alpha}-k$ is larger than $\pi_h^{pool2*} = \beta(1+\lambda d)$. So, we are left to compare between $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$ and $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$.

First, when $\frac{1-k}{(1-\alpha)(1+d)+1} \le \beta \le \frac{1}{(1-\alpha)(1+d)}$, $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ exists and $\pi_h^{sep1*} = \beta(1+d)$, so $\pi_h^{sep1*} > \pi_h^{sep3*}$ leads to $\beta > \frac{1-\alpha+k\alpha}{(1-\alpha)(1+d)-\alpha}$. Second, when $\frac{1}{(1-\alpha)(1+d)} < \beta \le \hat{\beta}_3$, $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ exists and $\pi_h^{sep1*} = \frac{1}{1-\alpha}$, where

$$\hat{\beta}_3 \equiv \begin{cases} \frac{\alpha+(1-\alpha)k}{(1-\alpha)(\alpha(1+d)-1)} & \text{if } \alpha(1+d) > 1, \\ 1-k & \text{if } \alpha(1+d) \le 1. \end{cases}$$

In this case, we have $\pi_h^{sep1*} > \pi_h^{sep3*}$.

Combining these two cases, $\pi_h^{sep1*} > \pi_h^{sep3*}$ when $\min\left\{\max\{\frac{1-k}{(1-\alpha)(1+d)+1}, \frac{1-\alpha+k\alpha}{(1-\alpha)(1+d)-\alpha}\}, \frac{1}{(1-\alpha)(1+d)}\right\} < \beta \le \hat{\beta}_3$. Therefore, in Region 2, the SUE is $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ if and only if $\beta \in \hat{\mathcal{P}}_{R2}$, where $\hat{\mathcal{P}}_{R2} = (\hat{\beta}_1, \hat{\beta}_2) \cap \left(\min\left\{\max\{\frac{1-k}{(1-\alpha)(1+d)+1}, \frac{1-\alpha+k\alpha}{(1-\alpha)(1+d)-\alpha}\}, \frac{1}{(1-\alpha)(1+d)}\right\}, \hat{\beta}_3\right]$ and $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$ if and only if $\beta \in (\hat{\beta}_1, \hat{\beta}_2) \setminus \left(\min\left\{\max\{\frac{1-k}{(1-\alpha)(1+d)+1}, \frac{1-\alpha+k\alpha}{(1-\alpha)(1+d)-\alpha}\}, \frac{1}{(1-\alpha)(1+d)}\right\}, \hat{\beta}_3\right]$.

Region 3: $k \le \max\{k_1, k_2\}$. With respect to β, this region is equivalent to $\beta \ge \hat{\beta}_2$. We further divide this region into two cases based on whether the $pool5$-type PBE exists or not: $\lambda \ge \hat{\lambda}^{imp}$ and $\lambda < \hat{\lambda}^{imp}$, where $\hat{\lambda}^{imp} \equiv \max\{\hat{\lambda}_1, \hat{\lambda}_2\}$, $\hat{\lambda}_1 = \frac{\alpha(1+d)-1}{d}$, and $\hat{\lambda}_2 = \frac{\alpha}{(1-\alpha)d}$.

Case 1: $\lambda \ge \hat{\lambda}^{imp}$. In this case, the $pool5$-type PBE exists; also, $\hat{\beta}_2$ is reduced to $\hat{\beta}_2 = \frac{k}{(1-\alpha)(1+\lambda d)-1}$ because $k_1 \ge k_2$ always holds. Moreover, $(\sigma_h^{pool5*}, \sigma_l^{pool5*})$ dominates $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$ and $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$, because $\pi_h^{pool5*} = 1-\beta+\beta(1+\lambda d)-k$ is larger than $\pi_h^{sep3*} = 1-\beta+\frac{\beta+k}{1-\alpha}-k$ and $\pi_h^{pool2*} = \beta(1+\lambda d)$. Therefore, we are left to compare between $(\sigma_h^{pool5*}, \sigma_l^{pool5*})$ and $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$.

We discuss three subcases. First, when $\beta \ge \frac{1}{(1-\alpha)(1+\lambda d)}$, $(\sigma_h^{pool5*}, \sigma_l^{pool5*})$ dominates $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$, because $\pi_h^{pool5*} = 1-\beta+\beta(1+\lambda d)-k$ is larger than $\pi_h^{sep1*} = \frac{1}{1-\alpha}$. Second, when $\frac{1}{(1-\alpha)(1+d)} < \beta < \frac{1}{(1-\alpha)(1+\lambda d)}$, $\pi_h^{sep1*} = \frac{1}{1-\alpha} > \pi_h^{pool5*}$ leads to $\beta < \frac{k+\alpha(1-k)}{(1-\alpha)\lambda d}$; note that under this condition, $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ exists. Third, when $\beta \le \frac{1}{(1-\alpha)(1+d)}$, $\pi_h^{sep1*} = \beta(1+d) > \pi_h^{pool5*}$ leads to $\beta > \frac{1-k}{1+(1-\lambda)d}$; note also that under this condition, $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ exists. Together, $\pi_h^{sep1*} > \pi_h^{pool5*}$ when $\min\left\{\frac{1}{(1-\alpha)(1+d)}, \frac{1-k}{1+(1-\lambda)d}\right\} < \beta < \max\left\{\frac{k+\alpha(1-k)}{(1-\alpha)\lambda d}, \frac{1}{(1-\alpha)(1+d)}\right\}$.

Therefore, in Region 3, when $\lambda \ge \hat{\lambda}^{imp}$, the SUE is $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ if and only if $\beta \in \hat{\mathcal{P}}_{R3}$, where $\hat{\mathcal{P}}_{R3} = [\hat{\beta}_2, 1-k) \cap \left(\min\left\{\frac{1}{(1-\alpha)(1+d)}, \frac{1-k}{1+(1-\lambda)d}\right\}, \max\left\{\frac{k+\alpha(1-k)}{(1-\alpha)\lambda d}, \frac{1}{(1-\alpha)(1+d)}\right\}\right)$, and is $(\sigma_h^{pool5*}, \sigma_l^{pool5*})$ iff $\beta \in [\hat{\beta}_2, 1-k) \setminus \left(\min\left\{\frac{1}{(1-\alpha)(1+d)}, \frac{1-k}{1+(1-\lambda)d}\right\}, \max\left\{\frac{k+\alpha(1-k)}{(1-\alpha)\lambda d}, \frac{1}{(1-\alpha)(1+d)}\right\}\right)$.

Case 2: $\lambda < \hat{\lambda}^{imp}$. Note that we only need to consider the scenario where $\hat{\lambda}_1 > \hat{\lambda}_2$ such that $\hat{\lambda}^{imp}$ is reduced to $\hat{\lambda}_1$ and hence $\lambda < \hat{\lambda}_1$; otherwise, when $\hat{\lambda}_1 \le \hat{\lambda}_2 \Leftrightarrow \alpha(1-\alpha)(1+d)-1 \le 0$, Region 3 would be empty in this case. Given $\lambda < \hat{\lambda}_1 \Leftrightarrow 1+\lambda d < \alpha(1+d)$, we have $k_2 > k_1$ and thus $\hat{\beta}_2$ is reduced to $\hat{\beta}_2 = \frac{k}{\alpha(1-\alpha)(1+d)-1}$. Therefore, neither $sep3$-type PBE nor $pool5$-type PBE exists. Also, $pool2$-type PBE does not exist, as $1-\beta+\beta\alpha(1+d)-\beta(1+\lambda d) > 1-\beta > k$. Therefore, only $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ could be the SUE. The existence condition of $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ is reduced to $\frac{1-k}{(1-\alpha)(1+d)+1} < \beta < \frac{\alpha+(1-\alpha)k}{(1-\alpha)(\alpha(1+d)-1)}$ as $\alpha(1+d) > 1+\lambda d \ge 1$.

Therefore, in Region 3, when $\lambda < \hat{\lambda}^{imp}$, the SUE is $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ if and only if $\beta \in \hat{\mathcal{P}}_{R4} = [\hat{\beta}_2, 1-k) \cap \left(\frac{1-k}{(1-\alpha)(1+d)+1}, \frac{\alpha+(1-\alpha)k}{(1-\alpha)(\alpha(1+d)-1)}\right)$ and does not exist if and only if $\beta \in [\hat{\beta}_2, 1-k) \setminus \left(\frac{1-k}{(1-\alpha)(1+d)+1}, \frac{\alpha+(1-\alpha)k}{(1-\alpha)(\alpha(1+d)-1)}\right)$.

Summary. Like the main model analysis, we can further combine Region 2 and Region 3 to conclude that the SUE is $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ when β is intermediate. That is, we can combine $\hat{\mathcal{P}}_{R2}$ and $\hat{\mathcal{P}}_{R3}$ (if neither is empty) or combine $\hat{\mathcal{P}}_{R2}$ and $\hat{\mathcal{P}}_{R4}$ (if neither is empty).

If neither $\hat{\mathcal{P}}_{R2}$ nor $\hat{\mathcal{P}}_{R3}$ is empty, then $\hat{\beta}_2 = \frac{k}{(1-\alpha)(1+\lambda d)-1}$ and the following must hold: (1) $\hat{\beta}_3 > \hat{\beta}_2$, otherwise $\hat{\mathcal{P}}_{R3}$ would be empty as $\hat{\beta}_3$ is the upper bound with respect to β for $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ to exist; (2) $\pi_h^{sep1*} > \pi_h^{sep3*}$ at $\beta = \hat{\beta}_2$, because $\hat{\mathcal{P}}_{R2}$ is not empty and $\pi_h^{sep1*} > \pi_h^{sep3*}$ at $\beta = \hat{\beta}_3$. Further, one can verify that $\pi_h^{sep3*} = \pi_h^{pool5*}$ at $\beta = \hat{\beta}_2$. Thus, we can combine $\hat{\mathcal{P}}_{R2}$ and $\hat{\mathcal{P}}_{R3}$ into a continuous interval $(\beta_1^{imp}, \beta_2^{imp})$ with respect to β, where β_1^{imp} is the lower bound of $\hat{\mathcal{P}}_{R2}$ and β_2^{imp} is the upper bound of $\hat{\mathcal{P}}_{R3}$. If neither $\hat{\mathcal{P}}_{R2}$ nor $\hat{\mathcal{P}}_{R4}$ is empty, we can also combine $\hat{\mathcal{P}}_{R2}$ and $\hat{\mathcal{P}}_{R4}$ into a continuous interval $(\beta_1^{imp}, \beta_2^{imp})$ with respect to β, where β_1^{imp} is the lower bound of $\hat{\mathcal{P}}_{R2}$ and β_2^{imp} is the upper bound of $\hat{\mathcal{P}}_{R4}$.

Thus, we conclude that the SUE is $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ when $\beta \in (\beta_1^{imp}, \beta_2^{imp})$. For notational consistency, let $\hat{\beta}^{imp} \equiv \hat{\beta}_2$. Then, outside $(\beta_1^{imp}, \beta_2^{imp})$, the SUE is $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$ when $\left(\{\beta \le \beta_1^{imp}\} \cup \{\beta \ge \beta_2^{imp}\}\right) \cap \{\beta < \hat{\beta}^{imp}\}$ and is $(\sigma_h^{pool5*}, \sigma_l^{pool5*})$ when $\left(\{\beta \le \beta_1^{imp}\} \cup \{\beta \ge \beta_2^{imp}\}\right) \cap \{\beta \ge \hat{\beta}^{imp}\} \cap \left\{\lambda \ge \hat{\lambda}^{imp}\right\}$. There is no pure-strategy PBE when $\left(\{\beta \le \beta_1^{imp}\} \cup \{\beta \ge \beta_2^{imp}\}\right) \cap \{\beta \ge \hat{\beta}^{imp}\} \cap \left\{\lambda < \hat{\lambda}^{imp}\right\}$; we do not consider this parameter region for the extension analysis. We summarize the results in Table B2.

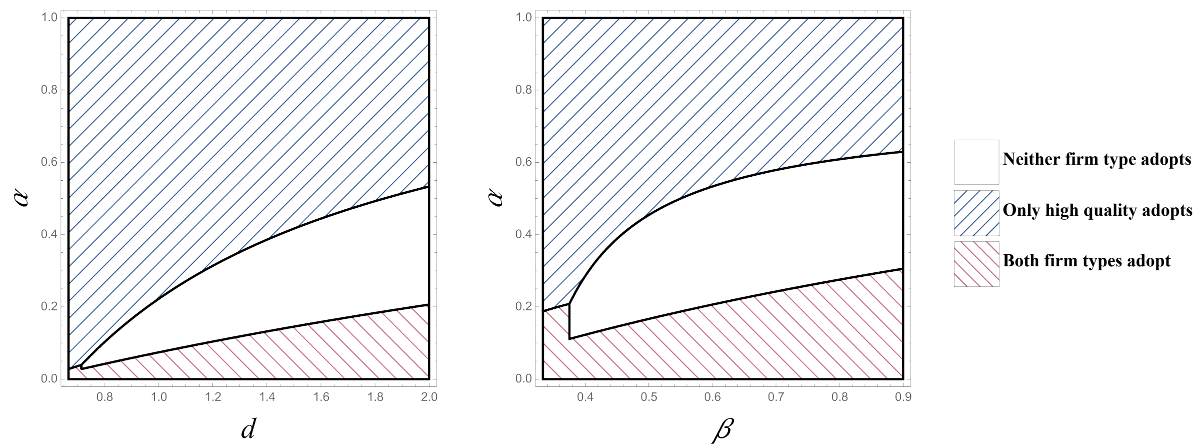
Table B2. Equilibrium outcomes under imperfect targeting

Condition		Equilibrium type	Equilibrium prices
$\beta_1^{imp} < \beta < \beta_2^{imp}$		No adoption $w_h^* = w_l^* = 0$	$p_h^* = \min\{1+d, \frac{1}{\beta(1-\alpha)}\}$ $p_l^* = 1$
$\beta \le \beta_1^{imp}$ or $\beta \ge \beta_2^{imp}$	$\beta < \hat{\beta}^{imp}$	Partial adoption $w_h^* = 1, w_l^* = 0$	$P_h^*(S) = \min\{\frac{\beta+k}{\beta(1-\alpha)}, 1+d\}, P_h^*(N) = 1$ $p_l^* = 1$
	$\beta \ge \hat{\beta}^{imp}$ and $\lambda \ge \hat{\lambda}^{imp}$	Universal adoption $w_h^* = w_l^* = 1$	$P_h^*(S) = 1+\lambda d, P_h^*(N) = 1$ $P_l^*(S) = 1+\lambda d, P_l^*(N) = 1$
	$\beta \ge \hat{\beta}^{imp}$ and $\lambda < \hat{\lambda}^{imp}$	N/A	N/A

We rule out the no-pure-strategy region (the last row of Table B2) by restricting attention to $d \le 3$, a sufficient condition for a pure-strategy PBE to exist. Recall that the no-pure-strategy region lies within Region 3 discussed above, which is defined by $k \le \max\{k_1, k_2\}$. Under $d \le 3$, we have $\alpha(1-\alpha)(1+d) \le 1$ (using $\alpha(1-\alpha) \le 1/4$), which yields $\hat{\lambda}_1 \le \hat{\lambda}_2$ and $k_2 \le 0$. The inequality $\hat{\lambda}_1 \le \hat{\lambda}_2$ further implies $\hat{\lambda}^{imp} = \max\{\hat{\lambda}_1, \hat{\lambda}_2\} = \hat{\lambda}_2$, so the condition $\lambda < \hat{\lambda}^{imp}$ becomes $\lambda < \hat{\lambda}_2$, which yields $k_1 \le 0$. Hence $\max\{k_1, k_2\} \le 0 < k$, contradicting Region 3 and ruling out the no-pure-strategy region.

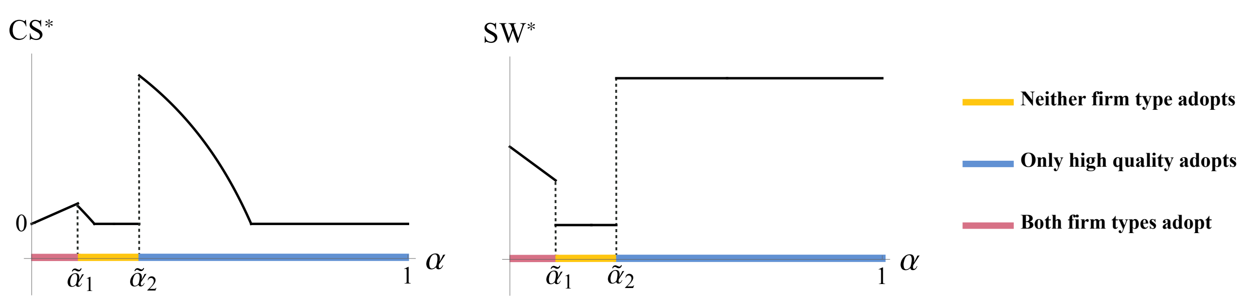
Similar to the main model analysis, one can also translate the conditions with respect to α or d. In summary, the high-quality firm will not adopt the PP technology when the number of informed consumers is intermediate ($\alpha_1^{imp} < \alpha < \alpha_2^{imp}$), when the size of quality-sensitive consumers is intermediate ($\beta_1^{imp} < \beta < \beta_2^{imp}$), and when the additional valuation for the high-quality product's premium features is moderate ($d_1^{imp} < d < d_2^{imp}$). Conditional on the high-quality firm's adoption, the low-quality firm will also adopt it when α is low, and when β and d are high. Figure B1 provides the graphical illustration. Moreover, Figure B2 shows that both the ex ante consumer surplus and social welfare may decrease with the number of informed consumers α.

Figure B1. Equilibrium adoption of the PP technology under imperfect targeting



(a) $\beta = 0.6, \lambda = 0.3, k = 0.1$ (b) $d = 2, \lambda = 0.3, k = 0.1$

Figure B2. Impacts of quality uncertainty prevalence under imperfect targeting



(a) Consumer surplus (b) Social welfare

Note. $d = 2$, $\lambda = 0.3$, $k = 0.1$, and $\beta = 0.4$.

OB.2. Profit comparison with the main model

Proof of Proposition 5. We use superscripts $per*$ and $imp*$ to indicate the high-quality firm's equilibrium profit in the perfect-targeting and imperfect-targeting scenarios, respectively. To prove Proposition 5, we focus on deriving a sufficient condition $\bar{\alpha}_1 < \alpha < \bar{\alpha}_2$ such that $\pi_h^{imp*} > \pi_h^{per*}$; outside $(\bar{\alpha}_1, \bar{\alpha}_2)$, there may be other α conditions under which $\pi_h^{imp*} > \pi_h^{per*}$.

Specifically, in the sufficient condition we derive, $\bar{\alpha}_1 = \max\{\frac{\beta d-k}{\beta+\beta d}, \frac{\beta d\lambda-k}{\beta d\lambda}, \frac{\beta(1+d)-1}{\beta d}\}$ and $\bar{\alpha}_2 = \frac{\beta d-k}{\beta d}$. As we will show, this condition implies that (1) the equilibrium in the imperfect-targeting scenario satisfies $w_h^* = 1$, $w_l^* = 0$, $p_l^* = 1$, $P_h^*(N) = 1$, and $P_h^*(S) = 1+d$, thereby $\pi_h^{imp*} = 1-\beta+\beta(1+d)-k$; (2) the equilibrium in the perfect-targeting scenario satisfies $w_h^* = 1$, $w_l^* = 0$, $p_l^* = 1$, $P_h^*(N, I) = P_h^*(N, U) = 1$, $P_h^*(S, I) = 1+d$, and $P_h^*(S, U) = 1+\frac{k}{\beta(1-\alpha)}$, thereby $\pi_h^{per*} = 1-\beta+\beta\alpha(1+d)+\beta(1-\alpha)(1+\frac{k}{\beta(1-\alpha)})-k$; (3) $\pi_h^{imp*} > \pi_h^{per*}$.

First, since $\alpha > \frac{\beta d-k}{\beta+\beta d}$, one can readily see from Step (vi) in the equilibrium analysis for the imperfect-targeting scenario that the equilibrium satisfies $w_h^* = 1$, $w_l^* = 0$, $p_l^* = 1$, $P_h^*(N) = 1$, and $P_h^*(S) = 1+d$ when $\bar{\alpha}_1 < \alpha < \bar{\alpha}_2$.

Second, recall that when $\frac{\beta d\lambda-k}{\beta d\lambda} < \alpha < \frac{\beta d-k}{\beta d}$, the equilibrium in the perfect-targeting scenario is derived by comparing $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ and $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$ (see Step (v) in the equilibrium analysis for the main model). Moreover, $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ is dominated by $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$ when $\alpha > \frac{\beta(1+d)-1}{\beta d}$. Therefore, the equilibrium in the perfect-targeting scenario satisfies $w_h^* = 1$, $w_l^* = 0$, $p_l^* = 1$, $P_h^*(N, I) = P_h^*(N, U) = 1$, $P_h^*(S, I) = 1+d$, and $P_h^*(S, U) = 1+\frac{k}{\beta(1-\alpha)}$ (which is smaller than $1+d$) when $\bar{\alpha}_1 < \alpha < \bar{\alpha}_2$.

Lastly, putting things together, we show $\pi_h^{imp*} > \pi_h^{per*}$ when $\bar{\alpha}_1 < \alpha < \bar{\alpha}_2$.

Moreover, in the above comparison, the low-quality firm's equilibrium profit is 1 in both the perfect-targeting and imperfect-targeting scenarios. Therefore, under the same condition, the firm's ex ante equilibrium profit is also lower in the perfect targeting scenario than in the imperfect targeting scenario. □

Proof of Corollary 2. We use superscripts $per*$ and $imp*$ to indicate the high-quality firm's equilibrium profit in the perfect-targeting and imperfect-targeting scenarios, respectively. To prove Corollary 2, we show a sufficient condition such that (1) the equilibrium in the imperfect-targeting scenario satisfies $w_h^* = 1$, $w_l^* = 0$, $p_l^* = 1$, $P_h^*(N) = 1$, and $P_h^*(S) = \min\{\frac{\beta+k}{\beta(1-\alpha)}, 1+d\}$, thereby $\pi_h^{imp*} = 1-\beta+\beta\min\{\frac{\beta+k}{\beta(1-\alpha)}, 1+d\}-k$; (2) the equilibrium in the perfect-targeting scenario satisfies $w_h^* = 0$, $w_l^* = 0$, $p_l^* = 1$, $P_h^* = 1+d$, thereby $\pi_h^{per*} = \beta(1+d)$; (3) $\pi_h^{imp*} > \pi_h^{per*}$.

First, we consider $\max\{\hat{\alpha}^{imp}, \alpha_2^{imp}\} < \alpha < 1$, in which case we have the partial-adoption equilibrium in the imperfect-targeting scenario with $w_h^* = 1$, $w_l^* = 0$, $p_l^* = 1$, $P_h^*(N) = 1$, and $P_h^*(S) = \min\{\frac{\beta+k}{\beta(1-\alpha)}, 1+d\}$. Second, we consider $\max\{\alpha_1, \frac{\beta(1+d)-1}{\beta(1+d)}\} < \alpha < \alpha_2$, in which case we have the no-adoption equilibrium in the perfect-targeting scenario with $w_h^* = 0$, $w_l^* = 0$, $p_l^* = 1$, $P_h^* = 1+d$. Lastly, we consider $\alpha > \frac{\beta(1+d)-1}{\beta(1+d)+k+\beta-1}$, under which $\pi_h^{imp*} > \pi_h^{per*}$ holds.

As an illustrative numerical example, in Figure 6, the above three conditions are correspondingly: (1) $\max\{0.102564, 0.222222\} < \alpha < 1$, (2) $\max\{0.113076, 0.166667\} < \alpha < 0.285714$, and (3) $\alpha > 0.222222$. Putting together, the result holds when $0.222222 < \alpha < 0.285714$. □

Online Appendix C: Using list prices together with personalized prices

Proof of Proposition 6 In this extended model, a firm's action σ can take the form of either $(w=0,p)$ or $(w=1,P(\theta,\tau),\bar{p})$. We follow the notations in the main model and use $\bar{p}(\sigma)$ to denote the list price used in σ, if any. If the firm adopts a list price, an uninformed consumer i's observed information will be $\gamma_i(\sigma)=(w(\sigma)=1,P(\sigma,\theta_i,\tau_i),\bar{p}(\sigma))$. Again, following the main model, it is without loss of generality to assume the pessimistic off-equilibrium beliefs: $\phi(\gamma_i(\sigma))=0$ if $\gamma_i(\sigma)\notin\{\gamma_i(\sigma_h),\gamma_i(\sigma_l)\}$.

In this extended model, we still have six potential equilibrium cases, as we discussed for the main model (see Table A1 in Online Appendix A). Moreover, our analysis below will show that the firm will not use a list price in equilibrium and that the equilibrium outcomes in the extended model will be identical to those in the main model where the firm cannot use list prices.

Similar to the main analysis, one can readily rule out Cases 5 and 6. In addition, one can show that $(\sigma_h^{sep1*},\sigma_l^{sep1*})$ in Case 1 and $(\sigma_h^{pool2*},\sigma_l^{pool2*})$ in Case 2 in the main model remain unchanged in this extended model. The previous analysis of the firm's IC constraints is unchanged, because when the firm deviates to a σ' with $w(\sigma')=1$, it will not use a list price together with personalized prices (i.e., $\bar{p}(\sigma')=\varnothing$) since uninformed consumers already believe that the product has low quality with probability one under the deviation. For ease of notations, we still use $(\sigma_h^{sep1*},\sigma_l^{sep1*})$ and $(\sigma_h^{pool2*},\sigma_l^{pool2*})$.

Next, we examine Cases 3 and 4 and show the results about a firm's usage of list price when it adopts PP.

First, in any $sep3$-type PBE where only the high-quality firm adopts the PP technology, the high-quality firm will not use a list price together with personalized prices. The reason is that in equilibrium uninformed consumers can already tell the product quality from w, so the high-quality firm will be more profitable by not using the list price. As a result, $(\sigma_h^{sep3*},\sigma_l^{sep3*})$ *in the main model, with the amendment that the high-quality firm does not use a list price (i.e.,* $\bar{p}_h=\varnothing$*), will still be the high-type-best* $sep3$*-type PBE in the extended model*. For ease of notation, we still use $(\sigma_h^{sep3*},\sigma_l^{sep3*})$.

Second, we will use Claims C1 and C2 below to show that $(\sigma_h^{sep4*},\sigma_l^{sep4*})$ *in the main model, with the amendment that neither firm type uses a list price (i.e.,* $\bar{p}_h=\bar{p}_l=\varnothing$*), will still be the high-type-best* $sep4$*-type PBE in the extended model*. For ease of notation, we still use $(\sigma_h^{sep4*},\sigma_l^{sep4*})$.

Given the above results, we can conclude that in any equilibrium, the firm will not use a list price, and that the equilibrium outcomes are identical to those in the main model where the firm cannot use list prices.

Claim C1. *The high-type-best* $sep4$*-type PBE* $(\sigma_h^{sep4*},\sigma_l^{sep4*})$ *in the main model, with the amendment that neither firm type uses a list price, constitutes an* $sep4$*-type PBE in the extended model, specifically,* $\sigma_h^{sep4*}=(w_h^{sep4*}=1,P_h^{sep4*}(S,I)=1+d,P_h^{sep4*}(N,I)=1,P_h^{sep4*}(S,U)=1+\lambda d,P_h^{sep4*}(N,U)=1,\bar{p}_h=\varnothing)$ *and* $\sigma_l^{sep4*}=(w_l^{sep4*}=1,P_l^{sep4*}(S,I)=1,P_l^{sep4*}(N,I)=1,P_l^{sep4*}(S,U)=1+\lambda d,P_l^{sep4*}(N,U)=1,\bar{p}_l=\varnothing)$.

Proof: To prove the claim, we show that the firm will not deviate from the action in the proposed PBE. First, the firm will not deviate from σ_j^{sep4*} to a σ' that still has $\bar{p}(\sigma')=\varnothing$ or does not involve $\bar{p}$ (when $w(\sigma')=0$).

The proof follows the analysis of finding $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ for the main model. Second, if the firm deviates to a σ' with $\bar{p}(\sigma') < 1 + d$ (which is the additional deviation compared to the main model analysis), all uninformed consumers will infer that the firm is low quality. Then, the firm's deviation profit must be lower than π_j^{sep4*}, because $P_j(\sigma', S, U) \leq 1 < P_j^*(S, U) = 1 + \lambda d$ and $P_h(\sigma', S, I) \leq \bar{p}(\sigma') < P_h^*(S, I) = 1 + d$. Hence, neither firm type will deviate to such σ'. Therefore, the proposed $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ constitutes an $sep4$-type PBE in this extended model. □

Claim C2. *$(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ in Claim C1 is the high-type-best $sep4$-type PBE in the extended model.*

Proof: Note that in any $sep4$-type PBE (where $w_h = w_l = 1$), it holds that the two firm types use the same list price: $\bar{p}_h = \bar{p}_l$. This is because otherwise uninformed consumers would infer the quality type from $\bar{p}$ and thus the low-quality firm would profitably deviate to not using the PP technology. Then, to prove this claim, we only need to show that any $sep4$-type PBE with $\bar{p}_h = \bar{p}_l < 1 + d$ must be defeated by the proposed $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ in Claim C1 where $\bar{p}_h = \bar{p}_l = \varnothing$. First, any $sep4$-type PBE with $\bar{p}_h = \bar{p}_l < 1 + d$ must be individually pooling for uninformed quality-sensitive consumers. The proof is similar to the main model analysis. Thus, in any $sep4$-type PBE with $\bar{p}_h = \bar{p}_l < 1 + d$, the personalized price for uninformed quality-sensitive consumers $P_h(S, U) \leq P_h^*(S, U)$. Second, the high-quality firm will charge informed quality-sensitive consumers $P_h(S, I) \leq \bar{p}_h < P_h^*(S, I) = 1 + d$ in any $sep4$-type PBE with $\bar{p}_h = \bar{p}_l < 1 + d$. Therefore, the high-quality firm's equilibrium profit in any $sep4$-type PBE with $\bar{p}_h = \bar{p}_l < 1 + d$ must be lower than π_h^{sep4*}. It follows that the proposed $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ in Claim C1 is the *high-type-best $sep4$*-type PBE in this extended model. □

Online Appendix D: Some consumers do not observe the adoption of PP

In the main model, we assume that all consumers can discern whether the firm adopts the PP technology, i.e., w. In practice, consumers may lack the experience or knowledge to identify the firm's pricing strategy, or the firm may not disclose the adoption very transparently. This extension considers the case where some consumers do not know about the adoption of PP technology and shows the robustness of our main findings.

We assume that a proportion η of consumers are *PP-naive* and they are ignorant about the adoption of PP technology (i.e., they do not know w), while the remaining proportion $1-\eta$ of consumers are *PP-savvy* and they know whether the firm adopts the PP technology (i.e., they know w). In other words, a PP-naive consumer does not know whether the price she receives is personalized or not and thus updates her belief about product quality only based on the received price. Specifically, if the firm does not adopt the PP technology, a PP-naive consumer's observed information is $\gamma_i(\sigma) = p(\sigma)$; if the firm uses its PP technology, her observed information is $\gamma_i(\sigma) = P(\sigma, \theta_i, \tau_i)$. Thus, for a PP-naive consumer, it is possible that $\gamma_i(\sigma_1) = \gamma_i(\sigma_2)$ even though $w(\sigma_1) \neq w(\sigma_2)$. By contrast, a PP-savvy consumer's observed information is the same as in the main model. The firm does not know whether a consumer is PP-naive or PP-savvy. Other model settings remain unchanged as in the main model. Similar to the analysis for the main model, we solve for the SUE, and we assume the pessimistic beliefs: $\phi(\gamma_i(\sigma)) = 0$ if $\gamma_i(\sigma) \notin \{\gamma_i(\sigma_h), \gamma_i(\sigma_l)\}$.

Compared to the main model analysis, a key difference is that even if the firm plays an off-equilibrium action $\sigma' \notin \{\sigma_h, \sigma_l\}$ with $w(\sigma') \neq w(\sigma_j)$, an uninformed PP-naive consumer can still use the Bayes' rule to update her belief as long as her observed information (the price she receives) remains unchanged, i.e., $\gamma_i(\sigma') = \gamma_i(\sigma_h)$ or $\gamma_i(\sigma') = \gamma_i(\sigma_l)$. Below, we will prove Claim D1, which shows that the equilibrium outcomes are identical to those in the main model when $\eta < \frac{1}{1+d}$. We focus on this parameter region for analysis simplicity. When $\eta > \frac{1}{1+d}$, it is also possible that our findings are robust.

Claim D1. *When the number of PP-naive consumers $\eta < \frac{1}{1+d}$, the equilibrium outcomes are identical to those in the main model (Table 2).*

Proof: To prove this claim, we will revisit each case in Table A1 in the main model and show that the PBE outcome in each case remains unchanged when $\eta < \frac{1}{1+d}$. Specifically, we will show that (1) the respective high-type-best PBE in Cases 1-4 (i.e., $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$, $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$, $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$, and $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$) remains unchanged, and that (2) no PBE exists in Cases 5 and 6. Thus, the SUE outcome remains identical as in Table 2 when $\eta < \frac{1}{1+d}$. The proof will focus on revisiting the IC constraints in the main model analysis.

(1.1) The high-type-best PBE in Case 1 $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ remains unchanged. To see this, we show that the IC constraints for both firm types derived in the main model analysis remain unchanged when $\eta < \frac{1}{1+d}$. Recall that in an $sep1$-type equilibrium, $w_h^{sep1} = w_l^{sep1} = 0$ and $\sigma_h^{sep1} \neq \sigma_l^{sep1}$.

First, we revisit the low-quality firm's IC constraint. The only difference from the main model analysis is that when the low-quality firm deviates to a σ' with $w(\sigma') = 1$, for uninformed quality-sensitive consumers, it could charge $P(\sigma', S, U) = 1$ to sell to all of them or charge $P(\sigma', S, U) = p_h^{sep1}$ to sell to only the PP-naive consumers among them. This is because these PP-savvy uninformed quality-sensitive consumers hold the belief $\phi(\gamma(\sigma')) = 0$ in such deviation regardless of $P(\sigma', S, U)$, whereas in the latter case these PP-naive uninformed quality-sensitive consumers have the same observed information $\gamma(\sigma') = \gamma(\sigma_h)$ and hence hold the belief $\phi(\gamma(\sigma')) = \phi(\gamma(\sigma_h)) = 1$. Thus, the low-quality firm's deviation profit from a σ' with $w(\sigma') = 1$ is $1-\beta+\beta\alpha+\beta(1-\alpha)\max\{1, \eta p_h^{sep1}\} - k$, rather than $1-k$ in the main model analysis. Since $p_h^{sep1} \leq 1+d$, it holds that $\max\{1, \eta p_h^{sep1}\} = 1$ when $\eta < \frac{1}{1+d}$, and thus the low-quality firm's IC constraint (IC_l^{sep1}) remains unchanged.

Next, we revisit the high-quality firm's IC constraint. Similarly, the only difference from the main model analysis is that when the high-quality firm deviates to a σ' with $w(\sigma') = 1$, for uninformed quality-sensitive consumers, it could charge $P(\sigma', S, U) = 1$ to sell to all of them or charge $P(\sigma', S, U) = p_h^{sep1}$ to sell to only the PP-naive consumers among them. Thus, the high-quality firm's deviation profit from a σ' with $w(\sigma') = 1$ is $1 - \beta + \beta\alpha(1 + d) + \beta(1 - \alpha)\max\{1, \eta p_h^{sep1}\} - k$, rather than $1 - \beta + \beta\alpha(1 + d) + \beta(1 - \alpha) - k$ in the main model analysis. Similarly, the high-quality firm's IC constraint (IC_h^{sep1}) also remains unchanged when $\eta < \frac{1}{1+d}$. Consequently, following the main model analysis, the high-type-best PBE in Case 1 $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ remains unchanged.

(1.2) The high-type-best PBE in Case 2 $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$ remains unchanged. To see this, one can also show that the IC constraints for both firm types derived in the main model analysis remain unchanged when $\eta < \frac{1}{1+d}$. Similar to the above discussion of Case 1, the difference from the main model analysis arises when the firm deviates to a σ' with $w(\sigma') = 1$. For similar reasons, one can show that (IC_l^{pool2}) and (IC_h^{pool2}) remain the same when $\eta < \frac{1}{1+d}$ and hence $(\sigma_h^{pool2*}, \sigma_l^{pool2*})$ remains unchanged.

(1.3) The high-type-best PBE in Case 3 $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$ remains unchanged. In an $sep3$-type PBE, $w_h^{sep3} = 1$ and $w_l^{sep3} = 0$, and $\sigma_h^{sep3} \neq \sigma_l^{sep3}$. Thus, in the main model analysis, uninformed quality-sensitive consumers must have different observed information from the two firm types' actions in equilibrium, i.e., $\phi(\gamma(\sigma_h)) \neq \phi(\gamma(\sigma_l))$. However, in this extended model, it is possible that PP-naive uninformed quality-sensitive consumers have the same observed information from the two firm types' actions, because they do not know $w(\sigma_j^{sep3})$. For example, this could occur when the low-quality firm chooses to charge $p_l^{sep3} = P_h^{sep3}(S, U) > 1$ to sell to only PP-naive uninformed quality-sensitive consumers, leading to a profit of $\beta(1 - \alpha)\eta P_h^{sep3}(S, U)$. That being said, when $\eta < \frac{1}{1+d}$, it is clear that the low-quality firm will choose not to do so in equilibrium. Instead, the low-quality firm will charge $p_l^{sep3} = 1$ to sell to all consumers, which leads to $\pi_l^{sep3} = 1$. This also implies $P_h^{sep3}(S, U) > 1$ in equilibrium.

Then, we can further show that $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$ in the main model continues to be the high-type-best $sep3$-type PBE in this extended model. Like the above discussion, we show that the IC constraints for both firm types derived in the main model analysis remain unchanged when $\eta < \frac{1}{1+d}$. First, one can readily verify that the low-quality firm's IC constraint (IC_l^{sep3}) remains unchanged. Second, we revisit the high-quality firm's IC constraint. The only difference from the main model analysis is that when the high-quality firm deviates to a σ' with $w(\sigma') = 0$ (which happens only when $P_h^{sep3}(S,U) < 1+d$), it could charge $p(\sigma') = 1$ to sell to all consumers, leading to a deviation profit of 1, charge $p(\sigma') = 1+d$ to sell to only informed quality-sensitive consumers, leading to a deviation profit of $\beta\alpha(1+d)$, or charge $p(\sigma') = P_h^{sep3}(S,U) > 1$ to sell to informed quality-sensitive consumers and PP-naive uninformed quality-sensitive consumers, leading to a deviation profit of $(\beta\alpha + \beta(1-\alpha)\eta)P_h^{sep3}(S,U)$. Clearly, it holds that $(\beta\alpha + \beta(1-\alpha)\eta)P_h^{sep3}(S,U) < \pi_h^{sep3} = \beta\alpha(1+d) + \beta(1-\alpha)P_h^{sep3}(S,U) + (1-\beta) - k$, and therefore the high-quality firm's IC constraint (IC_h^{sep3}) remains unchanged. Thus, the high-type-best PBE in Case 3 $(\sigma_h^{sep3*}, \sigma_l^{sep3*})$ remains unchanged.

(1.4) The high-type-best PBE in Case 4 $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ remains unchanged. In an $sep4$-type PBE, $w_h^{sep4} = w_l^{sep4} = 1$ and $\sigma_h^{sep4} \neq \sigma_l^{sep4}$. Recall that finding $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ in the main model analysis takes four steps. One can verify that the first two steps continue to hold. Still, an $sep4$-type equilibrium must be *individually pooling* for *uninformed quality-sensitive* consumers, thereby $P_h^{sep4}(S,U) = P_l^{sep4}(S,U) \leq 1 + \lambda d$. Next, we show that the IC constraints for both firm types derived in the third step remain unchanged when $\eta < \frac{1}{1+d}$. It follows that $(\sigma_h^{sep4*}, \sigma_l^{sep4*})$ remains unchanged.

First, we revisit the low-quality firm's IC constraint. The only difference from the main model analysis is that when the low-quality firm deviates to a σ' with $w(\sigma') = 0$, it could charge $p(\sigma') = 1$ to sell to all consumers, leading to a deviation profit of 1, or charge $p(\sigma') = P_l^{sep4}(S,U)$ to sell to only PP-naive uninformed quality-sensitive consumers, leading to a deviation profit of $\beta(1-\alpha)\eta P_l^{sep4}(S,U)$. When $\eta < \frac{1}{1+d}$, it holds that $\beta(1-\alpha)\eta P_l^{sep4}(S,U) < 1$, and hence the low-quality firm's IC constraint (IC_l^{sep4}) remains unchanged.

Second, we revisit the high-quality firm's IC constraint. The only difference from the main model analysis is that when the high-quality firm deviates to a σ' with $w(\sigma') = 0$, it could charge $p(\sigma') = 1$ to sell to all consumers, leading to a deviation profit of 1, charge $p(\sigma') = 1+d$ to sell to only informed quality-sensitive consumers, leading to a deviation profit of $\beta\alpha(1+d)$, or charge $p(\sigma') = P_h^{sep4}(S,U) > 1$ to sell to informed quality-sensitive consumers and PP-naive uninformed quality-sensitive consumers, leading to a deviation profit of $(\beta\alpha + \beta(1-\alpha)\eta)P_h^{sep4}(S,U)$. Since $(\beta\alpha + \beta(1-\alpha)\eta)P_h^{sep4}(S,U) < \pi_h^{sep4} = \beta\alpha(1+d) + \beta(1-\alpha)P_h^{sep4}(S,U) + (1-\beta) - k$, the high-quality firm's IC constraint (IC_h^{sep4}) remains unchanged.

(2) No PBE exists in Cases 5 and 6. First, no PBE exists in Case 5 where $w_h = w_l = 1$, because the two firm types will set different personalized prices for informed quality-sensitive consumers. Moreover, no PBE exists in Case 6 where $w_h = 0, w_l = 1$, because when $\eta < \frac{1}{1+d}$, even if the low-quality firm can charge uninformed quality-sensitive consumers personalized prices $p_h > 1$ to sell to the PP-naive ones among them, it cannot make up for the cost of adopting the PP technology. Thus, the low-quality firm will still deviate to not adopting the PP technology. $\square$

Online Appendix E: Quality-insensitive consumers have strictly higher WTP for the high-quality product than for the low-quality product

In the main model, we assume that a consumer i's willingness to pay for type-j firm's product is $v_{ij} = 1 + d$ if the consumer is quality-sensitive and the product has high quality, and is $v_{ij} = 1$ otherwise. We extend this setting such that (1) similar to a high-quality product, a low-quality product also provides different values to quality-sensitive and quality-insensitive consumers, and (2) like quality-sensitive consumers, quality-insensitive consumers are also willing to pay more for high quality than low quality.

Let q_j ($j \in \{h, l\}$) denote the product quality, either high or low, where $q_h > q_l > 0$. A consumer's quality preference can be low (θ_L) or high (θ_H), where $\theta_H > \theta_L > 0$. The size of high-preference consumers (denoted as H) is $0 < \beta < 1$ and the remaining $1 - \beta$ are low-preference consumers (denoted as L). Knowing quality q_j, a θ_i-type consumer's willingness to pay is given by $\theta_i q_j$. All the other settings remain unchanged. Without loss of generality, we normalize q_l to one and θ_L to one, and therefore $q_h > 1$ and $\theta_H > 1$.

In this extended setup, we make the following parameter assumptions throughout the analysis: $\beta > \max\{1/2, 1/\theta_H\}$ and $1 - \beta < k < (1 - \beta)(\lambda q_h + (1 - \lambda) q_l)$. The first condition suggests that the population of high-preference consumers is not too low, whereas the second condition suggests that the fixed adoption cost of the PP technology is neither too low nor prohibitively high. Under these assumptions, one can show that when all consumers are informed about product quality ($\alpha = 1$), the high-quality firm will adopt the PP technology, whereas the low-quality firm will not adopt it. The high-quality firm sets the personalized price for a consumer to her willingness to pay, whereas the low-quality firm sets the uniform price to θ_H and sells to only high-preference consumers. This result parallels the benchmark outcome in Lemma 1. Then, as in the main model analysis, we proceed to show that, in the presence of quality uncertainty, the firm's PP strategy significantly deviates from this perfect-information benchmark. The high-quality firm may refrain from adopting the PP technology, whereas the low-quality firm may choose to adopt it in equilibrium. Moreover, the equilibrium personalized price levels may depart from consumers' willingness to pay.

The derivation of the equilibrium takes a similar procedure as in the main model. First, we find the high-type-best PBE outcomes for the four cases $sep1$-type, $pool2$-type, $sep3$-type, and $sep4$-type, respectively. Then, we compare these PBE outcomes to derive the SUE outcome. To reduce the analysis and illustrate the robustness of our key insights, the PBE derivation focuses on $q_h \le \min\{4, \theta_H\}$, that is, the difference between high and low quality is not too large. Similar to Step (v) in the proof for the main model, after finding the high-type-best PBE outcomes, we can divide the discussion into three parameter regions to pin down the SUE. Let $k_1 = \beta\alpha\theta_H + (1-\beta)\alpha + \beta(1-\alpha)(\lambda\theta_H q_h + (1-\lambda)\theta_H) + (1-\beta)(1-\alpha)(\lambda q_h + (1-\lambda)) - \beta\theta_H$ and $k_2 = \beta\alpha\theta_H + (1-\beta)\alpha + \beta(1-\alpha)\theta_H q_h + (1-\beta)(1-\alpha)q_h - \beta\theta_H$.

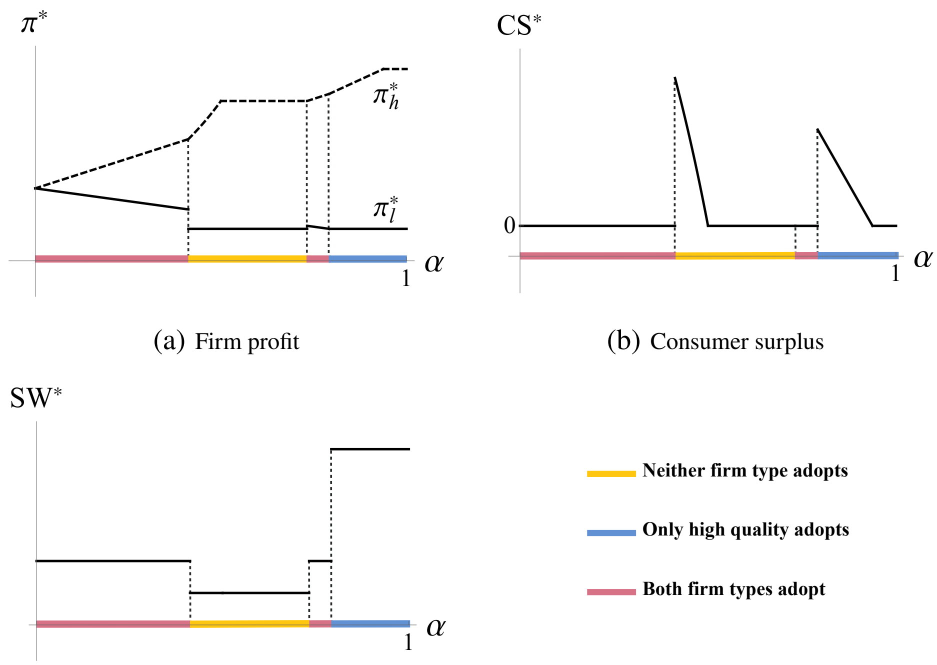
Region 1: $k \ge k_2$. In this region, as in the main model analysis, the SUE is a partial-adoption PBE where only the high-quality firm adopts the PP technology ($w_l^* = 0, w_h^* = 1$). Moreover, the low-quality firm sets the uniform price to $p_l^* = \theta_H$, whereas the high-quality firm sets the personalized price for a consumer to her willingness to pay, i.e., $P_h^*(H,I) = P_h^*(H,U) = \theta_H q_h$ and $P_h^*(L,I) = P_h^*(L,U) = q_h$. In this case, the equilibrium personalized price levels do not depart from consumers' willingness to pay.

Region 2: $k_1 < k < k_2$. In this region, as in the main model analysis, the SUE is either a no-adoption PBE or a partial-adoption PBE. In the no-adoption PBE, neither firm type adopts the PP technology ($w_l^* = 0, w_h^* = 0$); the high-quality firm sets the uniform price to $p_h^* = \min\{\frac{\theta_H}{1-\alpha}, \theta_H q_h\}$, whereas the low-quality firm sets the uniform price to $p_l^* = \theta_H$. In the partial-adoption PBE, only the high-quality firm adopts the PP technology ($w_l^* = 0, w_h^* = 1$); the low-quality firm sets the uniform price to $p_l^* = \theta_H$; the high-quality firm sets the personalized price for an informed consumer to her willingness to pay, i.e., $P_h^*(H,I) = \theta_H q_h$ and $P_h^*(L,I) = q_h$; however, the high-quality firm lowers the personalized price for an uninformed consumer from her willingness to pay level, i.e., $P_h^*(H,U) \leq \theta_H q_h$ and $P_h^*(L,U) < q_h$, which in particular satisfy $\beta\alpha\theta_H + (1-\beta)\alpha + \beta(1-\alpha)P_h^*(H,U) + (1-\beta)(1-\alpha)P_h^*(L,U) - k = \beta\theta_H$ such that the low-quality firm will not mimic the high-quality firm. Therefore, as in the main model analysis, the no-adoption PBE can dominate the partial-adoption PBE and be the SUE, in which case α is intermediate.

Region 3: $k \leq k_1$. In this region, as in the main model analysis, the SUE is either a no-adoption PBE or a universal-adoption PBE. In the no-adoption PBE, neither firm type adopts the PP technology ($w_l^* = 0, w_h^* = 0$); the high-quality firm sets the uniform price to $p_h^* = \min\{\frac{\theta_H}{1-\alpha}, \theta_H q_h\}$, whereas the low-quality firm sets the uniform price to $p_l^* = \theta_H$. In the universal-adoption PBE, both firm types adopt the PP technology ($w_l^* = 1, w_h^* = 1$); the two firm types charge the same personalized price for an uninformed consumer, i.e., $P_h^*(H,U) = P_l^*(H,U) = \lambda\theta_H q_h + (1-\lambda)\theta_H < \theta_H q_h$ and $P_h^*(L,U) = P_l^*(L,U) = \lambda q_h + (1-\lambda) < q_h$, whereas the personalized price for an informed consumer is equal to her willingness to pay, i.e., $P_h^*(H,I) = \theta_H q_h$, $P_h^*(L,I) = q_h$, $P_l^*(H,I) = \theta_H$, and $P_l^*(L,I) = 1$. As in the main model analysis, the no-adoption PBE can dominate the universal-adoption PBE and be the SUE, in which case α is intermediate.

Consistent with the main model, the high-quality firm refrains from adopting the PP technology when α is in an intermediate range. Conditional on the high-quality firm's adoption, the low-quality firm will also adopt the PP technology when α is relatively low. Figure E1 presents a numerical example. Specifically, we consider $q_h = 2$ and $\theta_H = 2$. So, θ_H-type consumers are willing to pay 4 and 2 for high and low quality, respectively; θ_L-type consumers are willing to pay 2 and 1 for high and low quality, respectively. Figure E1 exhibits a qualitatively similar pattern to that in Figure 4. As the number of informed consumers α increases, the equilibrium transitions from universal-adoption to no-adoption, then back to universal-adoption, and ultimately to partial-adoption. Also, the welfare implications can qualitatively hold. As α increases, the high-quality firm's equilibrium profit weakly increases; the low-quality firm's equilibrium profit can increase; the equilibrium consumer surplus can decrease; the equilibrium social welfare can decrease.

Figure E1. Equilibrium outcomes with heterogeneous willingness to pay



(a) Firm profit

(b) Consumer surplus

(c) Social welfare

Note. $q_h = 2$, $\theta_H = 2$, $\beta = 0.6$, $\lambda = 0.3$, and $k = 0.5$.

Online Appendix F: The high-quality firm incurs a higher marginal cost than the low-quality firm

In the main model, we assume that the marginal production cost is the same and normalized to zero, regardless of the firm's quality. We relax this assumption to follow the convention that the high-quality firm incurs a higher marginal cost than the low-quality firm. Specifically, we normalize the marginal cost of the low-quality firm to zero, whereas the marginal cost of the high-quality firm is $c > 0$. All the other settings remain unchanged. Moreover, we proceed to focus on $c < 1$; otherwise, the high-quality firm has no incentive to serve quality-insensitive consumers. With this extended setup, we also adjust the parameter assumptions in equation (3) to be $\beta(1+d-c) > 1-c$ and $k < (1-\beta)(1-c)$. These conditions have similar interpretations as in the main model.

To begin with, in the absence of consumer quality uncertainty, one can readily show the result in Lemma 1. Next, we examine the case with consumer quality uncertainty and show that our qualitative insights remain unchanged. The derivation of the equilibrium takes a similar procedure as in the main model. Below, we only highlight the high-type-best PBE outcomes for the four cases $sep1$-type, $pool2$-type, $sep3$-type, and $sep4$-type, respectively.

First, the $sep1$-type PBE exists when $k \geq 1+\beta\alpha d-\beta\min\{\frac{1}{(1-\alpha)\beta}, 1+d\}+\beta c-c$. Moreover, the high-type-best $sep1$-type equilibrium satisfies $\sigma_h^{sep1*} = (w_h^{sep1*} = 0, p_h^{sep1*} = \min\{\frac{1}{(1-\alpha)\beta}, 1+d\})$ and $\sigma_l^{sep1*} = (w_l^{sep1*} = 0, p_l^{sep1*} = 1)$. In this high-type-best equilibrium, $\pi_h^{sep1*} = \beta(\min\{\frac{1}{(1-\alpha)\beta}, 1+d\}-c)$ and $\pi_l^{sep1*} = 1$.

Second, the $pool2$-type equilibrium exists when $\beta \geq \frac{1}{(1-\alpha)(1+\lambda d)}$ and $k \geq 1+\beta\alpha d-\beta(1+\lambda d)+\beta c-c$. Moreover, the high-type-best $pool2$-type equilibrium satisfies $\sigma_h^{pool2*} = (w_h^{pool2*} = 0, p_h^{pool2*} = 1+\lambda d)$ and $\sigma_l^{pool2*} = (w_l^{pool2*} = 0, p_l^{pool2*} = 1+\lambda d)$. In this high-type-best equilibrium, $\pi_h^{pool2*} = \beta(1+\lambda d-c)$ and $\pi_l^{pool2*} = (1-\alpha)\beta(1+\lambda d)$.

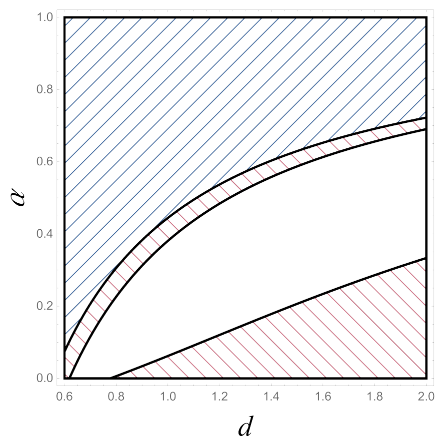
Third, the high-type-best $sep3$-type equilibrium satisfies $\sigma_l^{sep3*} = (w_l^{sep3*} = 0, p_l^{sep3*} = 1)$ and $\sigma_h^{sep3*} = (w_h^{sep3*} = 1, P_h^{sep3*}(S,I) = 1+d, P_h^{sep3*}(S,U) = \min\{1+\frac{k}{\beta(1-\alpha)}, 1+d\}, P_h^{sep3*}(N,I) = 1, P_h^{sep3*}(N,U) = 1)$. In this high-type-best equilibrium, $\pi_h^{sep3*} = \beta\alpha(1+d)+(1-\beta)+\beta(1-\alpha)\min\{1+\frac{k}{\beta(1-\alpha)}, 1+d\}-k-c$ and $\pi_l^{sep3*} = 1$.

Lastly, the $sep4$-type PBE exists when $k \leq \beta(1-\alpha)\lambda d$. Moreover, the high-type-best $sep4$-type equilibrium satisfies $\sigma_h^{sep4*} = (w_h^{sep4*} = 1, P_h^{sep4*}(S,I) = 1+d, P_h^{sep4*}(N,I) = 1, P_h^{sep4*}(S,U) = 1+\lambda d, P_h^{sep4*}(N,U) = 1)$ and $\sigma_l^{sep4*} = (w_l^{sep4*} = 1, P_l^{sep4*}(S,I) = 1, P_l^{sep4*}(N,I) = 1, P_l^{sep4*}(S,U) = 1+\lambda d, P_l^{sep4*}(N,U) = 1)$. In this high-type-best equilibrium, $\pi_h^{sep4*} = \beta\alpha(1+d)+\beta(1-\alpha)(1+\lambda d)+(1-\beta)-k-c$ and $\pi_l^{sep4*} = (1-\beta+\beta\alpha)+\beta(1-\alpha)(1+\lambda d)-k$.

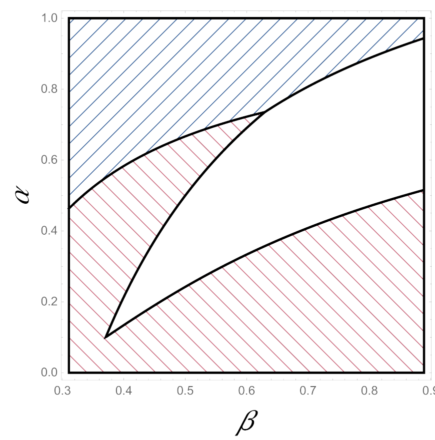
Then, we can compare these PBE outcomes to derive the SUE outcome. We find that the key results in the main model remain qualitatively unchanged. In particular, the high-quality firm should not adopt PP when an intermediate proportion of consumers know product quality (i.e., moderate α), and when a high-quality product, on average, offers moderate additional value to consumers compared to a low-quality product (i.e., moderate β and d). When the high-quality firm chooses to adopt the PP technology in equilibrium, the low-quality firm will also adopt it when α is relatively small, and when β and d are relatively large. As an illustration, Figure F1 shows the equilibrium pattern of PP adoption with $c = 0.1$ and $c = 0.3$. As a result of the firm's strategic decision, the counterintuitive welfare impacts regarding the key parameters also hold.

Figure F1. Equilibrium adoption of the PP technology under different marginal costs

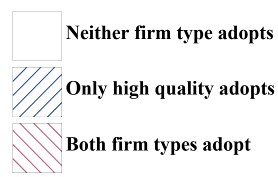
(1) The effect of α and d | **(2) The effect of α and β**

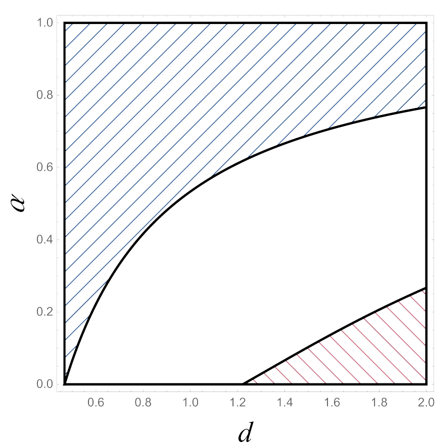


(a) $\beta = 0.6, \lambda = 0.3, k = 0.1, c = 0.1$

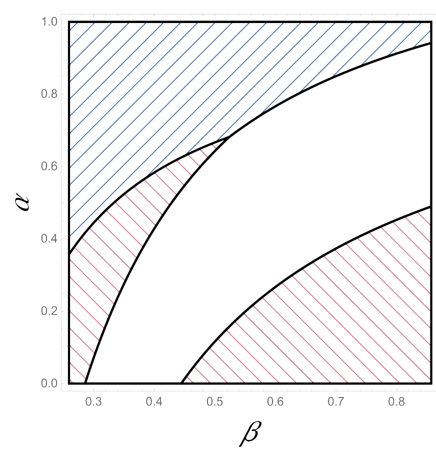


(c) $d = 2, \lambda = 0.3, k = 0.1, c = 0.1$





(b) $\beta = 0.6, \lambda = 0.3, k = 0.1, c = 0.3$



(d) $d = 2, \lambda = 0.3, k = 0.1, c = 0.3$

Online Appendix G: The firm can use advertising as a quality signaling tool

We consider an extension model where the firm can choose to advertise or not, in addition to the pricing decision. Specifically, the firm can incur a cost of $f > 0$ to advertise. We use $x \in \{1, 0\}$ to denote the advertising decision, with $x = 1$ being advertising and $x = 0$ being not. Then the firm's action σ can take the form of $(x, w = 0, p)$ or $(x, w = 1, P(\theta, \tau))$. As in Milgrom and Roberts (1986), the advertising has no direct impact on demand or gross profit, but only has a potential impact on uninformed quality-sensitive consumers' perceptions of quality. All the other setups and assumptions remain unchanged as in the main model. Following the main model, it is without loss of generality to assume the pessimistic off-equilibrium beliefs: $\phi(\gamma_i(\sigma)) = 0$ if $\gamma_i(\sigma) \notin \{\gamma_i(\sigma_h), \gamma_i(\sigma_l)\}$. To illustrate the robustness of our key insight, we show that in equilibrium, the high-quality firm may advertise *and* also refrain from adopting PP to signal its high quality. Therefore, our key signaling insight can remain relevant even when the firm uses advertising as a complementary signaling instrument. In particular, we prove the following claim.

Claim G1. *When β, d, and α are moderate, in equilibrium, the high-quality firm advertises and also refrains from adopting PP; specifically, $\sigma_h^* = (x_h^* = 1, w_h^* = 0, p_h^* = \frac{1+f}{\beta(1-\alpha)})$ and $\sigma_l^* = (x_l^* = 0, w_l^* = 0, p_l^* = 1)$.*

Proof: The proof takes the following seven steps. Step (1): We rule out any PBE where only the low-quality firm advertises $(x_h = 0, x_l = 1)$. Therefore, depending on the firm's advertising action, there are three possible PBE cases: neither firm type advertises $(x_h = x_l = 0)$, both firm types advertise $(x_h = x_l = 1)$, and only the high-quality firm advertises $(x_h = 1, x_l = 0)$. Step (2): We show that in any PBE where both firm types advertise $(x_h = x_l = 1)$, either both firm types adopt PP or neither firm type adopts PP. Step (3): We show that each high-type-best PBE we identified in the main model, with the *amendment* that neither firm type advertises $(x_h = x_l = 0)$, still constitutes a PBE in this extended model. This identifies the PBEs that yield the highest profit for the high-quality firm, among all the PBEs with $x_h = x_l = 0$. Step (4): We show that in any PBE where only the high-quality firm advertises $(x_h = 1, x_l = 0)$, the low-quality firm does not adopt PP and sets a uniform price of one. Then, there are two equilibrium cases, depending on whether the high-quality firm adopts PP. Step (5): For the equilibrium case where only the high-quality firm advertises $(x_h = 1, x_l = 0)$ and neither firm type adopts PP $(w_h = w_l = 0)$, we find the PBE that yields the highest profit for the high-quality firm. Step (6): For the equilibrium case where only the high-quality firm advertises $(x_h = 1, x_l = 0)$ and only the high-quality firm adopts PP $(w_h = 1, w_l = 0)$, we find the PBE that yields the highest profit for the high-quality firm. Step (7): We show that the PBE identified in step (5) can defeat all other possible PBEs and be the SUE.

Step (1): We first rule out any PBE where only the low-quality firm advertises $(x_h = 0, x_l = 1)$, because uninformed quality-sensitive consumers will tell the firm type based on the advertising action and therefore the low quality firm will deviate to not advertising and save the advertising cost.

Step (2): Similarly, we can rule out any PBE where both firm types advertise $(x_h = x_l = 1)$ but uninformed quality-sensitive consumers can tell the firm type from the pricing action $(\gamma_i(\sigma_h) \neq \gamma_i(\sigma_l))$, because the low-quality firm will deviate to not advertising and save the advertising cost. Therefore, in any PBE with $x_h = x_l = 1$, either both firm types adopt PP or neither firm type adopts PP, and such PBE must be (individually) pooling for uninformed quality-sensitive consumers.

Step (3): To prove that each high-type-best PBE we identified in the main model, with the amendment $x_h = x_l = 0$, still constitutes a PBE, we show that the firm will not deviate from the action in such proposed PBEs. First, the firm will not deviate to a σ' with $x(\sigma') = 0$, that is, the firm only deviates from the pricing action in a proposed PBE. The proof follows the analysis of finding the original PBE for the main model. Second, the firm will not deviate to a σ' with $x(\sigma') = 1$. Otherwise, given the deviation from the advertising action, uninformed quality-sensitive consumers will believe that the firm is low quality. Then, based on the first point, no matter whether the firm also deviates from the pricing action in the proposed PBE, the deviation profit must be lower than the equilibrium profit.

Step (4): It is straightforward that in any PBE where only the high-quality firm advertises ($x_h = 1, x_l = 0$), the low-quality firm must set a uniform price of one, because uninformed quality-sensitive consumers can tell the firm type from the advertising action. It follows that there are two equilibrium cases, depending on whether the high-quality firm adopts PP.

Step (5): One can show that when $\beta > \frac{1+f}{(1-\alpha)(1+d)}$ and $k \geq \beta\alpha(1+d) + 1 - \beta + \beta(1-\alpha) - \frac{1}{1-\alpha}$, $\sigma_h = (x_h = 1, w_h = 0, p_h = \frac{1+f}{\beta(1-\alpha)})$ and $\sigma_l = (x_l = 0, w_l = 0, p_l = 1)$ constitute a PBE, which is the high-type-best PBE for the equilibrium case where only the high-quality firm advertises ($x_h = 1, x_l = 0$) and neither firm type adopts PP ($w_h = w_l = 0$). The analysis is similar to finding the high-type-best $sep1$-type PBE in the main model by deriving the firm's IC constraints.

Step (6): One can show that when $f + k < \beta(1-\alpha)d$, $\sigma_h = (x_h = 1, w_h = 1, P_h(S,I) = 1 + d, P_h(S,U) = 1 + \frac{k+f}{\beta(1-\alpha)}, P_h(N,I) = 1, P_h(N,U) = 1)$ and $\sigma_l = (x_l = 0, w_l = 0, p_l = 1)$ constitute a PBE, which is the high-type-best PBE for the equilibrium case where only the high-quality firm advertises ($x_h = 1, x_l = 0$) and only the high-quality firm adopts PP ($w_h = 1, w_l = 0$). The analysis is similar to finding the high-type-best $sep3$-type PBE in the main model by deriving the firm's IC constraints.

Step (7): We show that when $\frac{1+f}{(1-\alpha)(1+d)} < \beta < \frac{1}{(1-\alpha)d}$ and $\beta(1-\alpha)\lambda d < k < \beta(1-\alpha)d$, the PBE identified in step (5), $\sigma_h^* = (x_h^* = 1, w_h^* = 0, p_h^* = \frac{1+f}{\beta(1-\alpha)})$ and $\sigma_l^* = (x_l^* = 0, w_l^* = 0, p_l^* = 1)$, constitutes the unique SUE for this extended model. The high-quality firm's equilibrium profit is $\pi_h^* = \beta\frac{1+f}{\beta(1-\alpha)} - f = \frac{1}{1-\alpha} + \frac{\alpha f}{1-\alpha}$. The associated conditions represent moderate ranges of β, d, or α. Note that they are nonempty provided $f < 1/d$.

To show the SUE, we first recall from the main model analysis that when $\frac{1}{(1-\alpha)(1+d)} < \beta < \frac{1}{(1-\alpha)d}$ and $\beta(1-\alpha)\lambda d < k < \beta(1-\alpha)d$, $(\sigma_h^{sep1*} = (w_h^{sep1*} = 0, p_h^{sep1*} = \frac{1}{\beta(1-\alpha)}), \sigma_l^{sep1*} = (w_l^{sep1*} = 0, p_l^{sep1*} = 1))$ is the SUE in the main model, in which case the high-quality firm's equilibrium profit is $\frac{1}{1-\alpha}$; in particular, we show that $(\sigma_h^{sep1*}, \sigma_l^{sep1*})$ defeats all the other high-type-best PBEs in the main model.

Note that all the PBEs identified in step (3) have $x_h = x_l = 0$ and thus yield the same firm profit as those corresponding PBEs in the main model without the amendment $x_h = x_l = 0$. Therefore, under the proposed conditions, the PBE identified in step (5) defeats all the PBEs identified in step (3) because the former yields higher profit for the high-quality firm: $\frac{1}{1-\alpha}+\frac{\alpha f}{1-\alpha}>\frac{1}{1-\alpha}$.

Furthermore, one can show that under the proposed conditions, the PBE identified in step (5) defeats the PBE identified in step (6), as the former yields higher profit for the high-quality firm: $\frac{1}{1-\alpha}+\frac{\alpha f}{1-\alpha}>\frac{1}{1-\alpha}$ and $\frac{1}{1-\alpha}>1+\beta\alpha d \Leftrightarrow \beta<\frac{1}{(1-\alpha)d}$. Note that $\frac{1+f}{(1-\alpha)(1+d)}<\beta \Rightarrow f+k<\beta(1-\alpha)d$ as $k<1-\beta<1-\beta(1-\alpha)$.

Lastly, under the proposed conditions, we can also eliminate any PBE with $x_h=x_l=1$ (i.e., both firm types advertise). As noted in step (2), such PBEs must be (individually) pooling for uninformed quality-sensitive consumers, and therefore the pricing actions in these PBEs are in the same manner as the $pool2$-type or $sep4$-type PBEs in the main model. Thus, with the advertising cost, the high-quality firm's profit in these PBEs must be lower than its profit in the PBEs that are formed by supplementing the high-type-best $pool2$-type or $sep4$-type PBE in the main model with the advertising actions $x_h=x_l=0$. Since we have ruled out the PBEs with $x_h=x_l=0$, we can also eliminate the PBEs with $x_h=x_l=1$. $\square$

Online Appendix H: The firm's cost of adopting PP is private information

In our baseline model, we assume that the firm's cost of adopting PP technology, k, is common knowledge. Here we show that our key insight about PP adoption qualitatively remains valid under a binary-cost specification in which the adoption cost is the firm's private information, so that the firm's type becomes two-dimensional: product quality and PP adoption cost.

Without loss of generality, we assume that the firm is high cost (k_H) with probability $1/2$ and low cost (k_L) with probability $1/2$, where $0 < k_L < k_H < 1-\beta$. The prior distributions of product quality and PP adoption cost are independent, so that consumers' (potential) inference about the firm's cost type does not, by itself, change their beliefs about quality. In practice, PP adoption costs largely reflect implementation frictions (e.g., data integration and engineering effort, vendor selection and contracting, and internal operation) that are not inherently tied to product quality, so they need not be systematically correlated with quality. Therefore, there are four firm types: (quality type, cost type) $= \{(h, k_H), (h, k_L), (l, k_H), (l, k_L)\}$, with prior probabilities $\frac{\lambda}{2}$, $\frac{\lambda}{2}$, $\frac{1-\lambda}{2}$, and $\frac{1-\lambda}{2}$, respectively. All the other setups remain unchanged as in the main model.

In this extended model, we will show that regardless of the cost type, the high-quality firm may still refrain from adopting PP to signal its high quality in equilibrium. In particular, we prove the following claim.

Claim H1. *When β, d, and α are moderate, in equilibrium, neither the high-quality firm nor the low-quality firm adopts PP, regardless of the cost type; specifically, $\sigma^*_{(h,k_H)} = \sigma^*_{(h,k_L)} = (w^* = 0, p^* = \frac{1}{\beta(1-\alpha)})$ and $\sigma^*_{(l,k_H)} = \sigma^*_{(l,k_L)} = (w^* = 0, p^* = 1)$.*

Proof: The proof proceeds as follows. First, we provide a partition of potential PBE cases in this extended model (Table H1). Then, we use the partition to help derive the proposed equilibrium.

As in the main model, the firm's action σ takes the form of either $(w = 0, p)$ or $(w = 1, P(\theta, \tau))$. Note that consumers' beliefs about the firm's cost type are payoff-irrelevant: the adoption costs affect the firm's incentives but do not enter consumers' utilities. As a result, only beliefs about product quality matter for demand. Consistent with the main model, we focus on how the observed pricing information shapes uninformed quality-sensitive consumers' posterior beliefs about quality, since they are the only consumer segment for which belief updating affects willingness to pay and hence the firm's equilibrium pricing incentives. Recall that we use $\phi(\gamma(\sigma))$ to represent their believed probability that the firm has high quality ($j = h$), based on their observed information $\gamma(\sigma)$. In particular, if the firm does not adopt PP, uninformed quality-sensitive consumers observe $w = 0$ and the uniform price: $\gamma(\sigma) = (w(\sigma) = 0, p(\sigma))$; if the firm adopts PP, they observe $w = 1$ and their own personalized price: $\gamma(\sigma) = (w(\sigma) = 1, P(\sigma, S, U))$.

In a pure-strategy PBE, each firm type's action induces a deterministic observed-information realization $\gamma(\sigma)$. Two firm types "pool" from an uninformed quality-sensitive consumer's perspective if and only if their actions induce the same information realization $\gamma(\sigma)$. Thus, any candidate PBE induces a partition of the four firm types into groups that may share some observed information. Given four firm types, there are 15 possible partitions, presented in Table H1, from all four types inducing an identical $\gamma(\sigma)$ to each type inducing a distinct $\gamma(\sigma)$. This framework is exhaustive: any pure-strategy equilibrium in this extended setting must logically fall into one of these 15 informational cases. For any "observed signal" $\gamma(\sigma)$ on the equilibrium path, uninformed quality-sensitive consumers use the Bayes' rule to form a posterior belief of high quality (i.e., $\phi(\gamma(\sigma))$) based on the composition of firm types within that specific partition. For example, in case 2, their posterior belief is computed as $\phi(\gamma_1) = \frac{\frac{\lambda}{2}+\frac{\lambda}{2}}{\frac{\lambda}{2}+\frac{\lambda}{2}+\frac{1-\lambda}{2}} = \frac{2\lambda}{1+\lambda}$ based on $\gamma_1 : \{(h,k_L),(h,k_H),(l,k_L)\}$. Following the main model analysis, it is without loss of generality to assume the pessimistic off-equilibrium beliefs: $\phi(\gamma(\sigma)) = 0$ if $\gamma(\sigma) \notin \{\gamma(\sigma_{(h,k_H)}), \gamma(\sigma_{(h,k_L)}), \gamma(\sigma_{(l,k_H)}), \gamma(\sigma_{(l,k_L)})\}$.

Table H1. Potential PBE cases by uninformed quality-sensitive consumers' observed information $\gamma(\sigma)$

Case	Partition of firm types by observed information $\gamma(\sigma)$	$\phi(\gamma_1)$	$\phi(\gamma_2)$	$\phi(\gamma_3)$	$\phi(\gamma_4)$
1	$\gamma_1 : \{(h,k_L),(h,k_H),(l,k_L),(l,k_H)\}$	λ	–	–	–
2	$\gamma_1 : \{(h,k_L),(h,k_H),(l,k_L)\}$; $\gamma_2 : \{(l,k_H)\}$	$\frac{2\lambda}{1+\lambda}$	0	–	–
3	$\gamma_1 : \{(h,k_L),(h,k_H),(l,k_H)\}$; $\gamma_2 : \{(l,k_L)\}$	$\frac{2\lambda}{1+\lambda}$	0	–	–
4	$\gamma_1 : \{(h,k_L),(l,k_L),(l,k_H)\}$; $\gamma_2 : \{(h,k_H)\}$	$\frac{\lambda}{2-\lambda}$	1	–	–
5	$\gamma_1 : \{(h,k_H),(l,k_L),(l,k_H)\}$; $\gamma_2 : \{(h,k_L)\}$	$\frac{\lambda}{2-\lambda}$	1	–	–
6	$\gamma_1 : \{(h,k_L),(h,k_H)\}$; $\gamma_2 : \{(l,k_L),(l,k_H)\}$	1	0	–	–
7	$\gamma_1 : \{(h,k_L),(l,k_L)\}$; $\gamma_2 : \{(h,k_H),(l,k_H)\}$	λ	λ	–	–
8	$\gamma_1 : \{(h,k_L),(l,k_H)\}$; $\gamma_2 : \{(h,k_H),(l,k_L)\}$	λ	λ	–	–
9	$\gamma_1 : \{(h,k_L),(h,k_H)\}$; $\gamma_2 : \{(l,k_L)\}$; $\gamma_3 : \{(l,k_H)\}$	1	0	0	–
10	$\gamma_1 : \{(l,k_L),(l,k_H)\}$; $\gamma_2 : \{(h,k_L)\}$; $\gamma_3 : \{(h,k_H)\}$	0	1	1	–
11	$\gamma_1 : \{(h,k_L),(l,k_L)\}$; $\gamma_2 : \{(h,k_H)\}$; $\gamma_3 : \{(l,k_H)\}$	λ	1	0	–
12	$\gamma_1 : \{(h,k_L),(l,k_H)\}$; $\gamma_2 : \{(h,k_H)\}$; $\gamma_3 : \{(l,k_L)\}$	λ	1	0	–
13	$\gamma_1 : \{(h,k_H),(l,k_L)\}$; $\gamma_2 : \{(h,k_L)\}$; $\gamma_3 : \{(l,k_H)\}$	λ	1	0	–
14	$\gamma_1 : \{(h,k_H),(l,k_H)\}$; $\gamma_2 : \{(h,k_L)\}$; $\gamma_3 : \{(l,k_L)\}$	λ	1	0	–
15	$\gamma_1 : \{(h,k_L)\}$; $\gamma_2 : \{(h,k_H)\}$; $\gamma_3 : \{(l,k_L)\}$; $\gamma_4 : \{(l,k_H)\}$	1	1	0	0

Based on the partition in Table H1, our subsequent analysis takes the following three steps. Step (1): We apply two conditions to rule out any PBE case that involves a low-quality type (at least one of the two cost types) pooling with a high-quality type (at least one of the two cost types) on some $\gamma(\sigma)$, i.e., cases 1-5, 7, 8, and 11-14. Step (2): We further eliminate cases 9 and 15 by showing $\sigma_{(l,k_H)} = \sigma_{(l,k_L)} = (w = 0, p = 1)$ in equilibrium. Step (3): We show that $\sigma_{(h,k_H)} = \sigma_{(h,k_L)} = (w = 0, p = \frac{1}{\beta(1-\alpha)})$ and $\sigma_{(l,k_H)} = \sigma_{(l,k_L)} = (w = 0, p = 1)$ can constitute a PBE and this proposed PBE can be the SUE, along with the conditions.

Step (1): First, note that a low-quality type can ensure a profit of 1 by setting a uniform price of 1 and serving the whole market. Then, we consider any PBE case that involves a low-quality type (at least one of the two cost types) pooling with a high-quality type (at least one of the two cost types) on some $\gamma^{pool}(\sigma)$ (see cases 1-5, 7, 8, and 11-14 in Table H1 for specifications), which can be ruled out if a low-quality type's profit by inducing $\gamma^{pool}(\sigma)$ is less than the reservation profit 1. Below we consider two possible scenarios regarding $\gamma^{pool}(\sigma)$.

If the pooled firm types do not adopt PP and therefore $\gamma^{pool}(\sigma) = (w = 0, p)$, then a low-quality type's profit is $\beta(1-\alpha)p$, where $1 < p \le 1 + \phi(\gamma^{pool}(\sigma))d$. Note that the highest posterior belief of high quality under these "pooled signals" is $\frac{2\lambda}{1+\lambda}$ (see Table H1), so we can rule out this scenario using the condition:

$$\beta < \frac{1}{(1-\alpha)(1+\frac{2\lambda}{1+\lambda}d)}.$$

If the pooled firm types adopt PP and therefore $\gamma^{pool}(\sigma) = (w = 1, P(S,U))$, then a low-quality type's profit is $1-\beta+\beta\alpha+\beta(1-\alpha)P(S,U)-k_L$ if it is the low-cost type, or $1-\beta+\beta\alpha+\beta(1-\alpha)P(S,U)-k_H$ if it is the high-cost type, where $1 < P(S,U) \le 1+\phi(\gamma^{pool}(\sigma))d$. Similarly, we can rule out this scenario using the condition (under which even the low-cost type finds it not profitable):

$$k_L > \beta(1-\alpha)\frac{2\lambda}{1+\lambda}d.$$

Step (2): Based on step (1), we know that a low-quality type must separate with a high-quality type for uninformed quality-sensitive consumers. Therefore, a low-quality type must set a uniform price of 1 in equilibrium and earn a profit of 1, i.e., $\sigma_{(l,k_H)} = \sigma_{(l,k_L)} = (w = 0, p = 1)$. This rules out cases 9 and 15.

Step (3): We first establish $\sigma_{(h,k_H)} = \sigma_{(h,k_L)} = (w = 0, p = \frac{1}{\beta(1-\alpha)})$ and $\sigma_{(l,k_H)} = \sigma_{(l,k_L)} = (w = 0, p = 1)$ as a PBE. Then we show that this proposed PBE can be the SUE.

Step (3.1): We establish the proposed PBE by showing that no firm type will deviate from the action in the PBE, if $\frac{1}{1-\alpha} > 1-\beta+\beta(1-\alpha)+\beta\alpha(1+d)-k_L$ and $\frac{1}{\beta(1-\alpha)} < 1+d$.

It is straightforward that a low-quality type will not deviate to any σ' with $w(\sigma') = 0$ and $p(\sigma') \neq \frac{1}{\beta(1-\alpha)}$ or any σ' with $w(\sigma') = 1$, since $\phi(\gamma(\sigma')) = 0$. Moreover, a low-quality type will not mimic a high-quality type by deviating to a σ' with $w(\sigma') = 0$ and $p(\sigma') = \frac{1}{\beta(1-\alpha)}$.

For a high-quality type, it will not deviate to any σ' with $w(\sigma') = 1$ because $\frac{1}{1-\alpha} > 1-\beta+\beta(1-\alpha)+\beta\alpha(1+d)-k_L$. Moreover, a high-quality type will not deviate to any σ' with $w(\sigma') = 0$ and $p(\sigma') \neq \frac{1}{\beta(1-\alpha)}$ because $\frac{1}{1-\alpha} > \max\{1, \beta\alpha(1+d)\}$.

Step (3.2): We show that the proposed PBE is the SUE, if $\frac{1}{1-\alpha} > 1+\beta\alpha d$ and $k_L < \beta(1-\alpha)d$.

We first note that among all possible PBEs with $w(\sigma_{(h,k_H)}) = 0$, $w(\sigma_{(h,k_L)}) = 0$, and $\sigma_{(l,k_H)} = \sigma_{(l,k_L)} = (w = 0, p = 1)$, the proposed PBE yields the highest profit for the high-quality firm, because $\frac{1}{\beta(1-\alpha)}$ is the highest possible uniform price a high-quality type can set to avoid mimicry given $\frac{1}{\beta(1-\alpha)} < 1+d$.

Next, we show that if a high-quality type adopts PP (i.e., $w(\sigma_{(h,k_H)}) = 1$ or $w(\sigma_{(h,k_L)}) = 1$), it cannot have higher profit than in the proposed PBE, given the condition $\frac{1}{1-\alpha} > 1+\beta\alpha d$. To prevent a low-quality type (especially the low-cost type) from mimicry, a high-quality type (regardless of its cost type) needs to limit its personalized price for uninformed quality-sensitive consumers to $P(S,U) \le 1+\frac{k_L}{\beta(1-\alpha)} (< 1+d)$ such that $1-\beta+\beta\alpha+\beta(1-\alpha)P(S,U)-k_L \le 1$ and therefore $1-\beta+\beta\alpha+\beta(1-\alpha)P(S,U)-k_H < 1$. Consequently, a high-quality type's profit cannot exceed $1-\beta+\beta\alpha(1+d)+\beta(1-\alpha)(1+\frac{k_L}{\beta(1-\alpha)})-k_L = 1+\beta\alpha d$.

In summary, case 10 is eliminated and the SUE corresponds to case 6. Claim H1 considers the following conditions:

$$\frac{1}{(1-\alpha)(1+d)} < \beta < \min\left\{\frac{1}{(1-\alpha)d}, \frac{1}{(1-\alpha)(1+\frac{2\lambda}{1+\lambda}d)}\right\} \text{ and } \beta(1-\alpha)\frac{2\lambda}{1+\lambda}d < k_L < \beta(1-\alpha)d.$$

These conditions represent moderate ranges of β, d, and α. □

Online Appendix I: Online experiment design and results

Online experiment design

Introduction:

Imagine you are shopping for [Product Name] on a company's website. After checking the available product information, you are still unsure about the product's quality.

Manipulation:

Uniform Pricing scenario: You know that the average price of wireless earbuds from other brands is approximately [Market Average Price]. The company's website lists its wireless earbuds at [Focal Product Price]. This price is not personalized and applies to all consumers.

Personalized Pricing scenario: You know that the average price of wireless earbuds from other brands is approximately [Market Average Price]. The company's website lists its wireless earbuds at [Focal Product Price]. This price is personalized for you based on your personal shopping information, and other consumers may receive different personalized prices for the same earbuds.

Questions:

Based on the price information, what do you think about the earbuds' quality compared to the average earbuds quality on the market? (1 Much lower quality than average, 2 Lower quality than average, 3 Slightly lower quality than average, 4 About average, 5 Slightly higher quality than average, 6 Higher quality than average, 7 Much higher quality than average)

Could you explain how you arrived at your assessment of the earbuds' quality based on the information you were given?

Attention checks:

In the survey, which type of price does the company charge for the earbuds?

In the survey, how much does the company charge for the earbuds?

In the survey, what is the average price of wireless earbuds from other brands?

Table I1. Online experiment result

Context (Focal vs. Market Avg. Price)	Uniform Pricing M (N)	Personalized Pricing M (N)	95% CI of Mean Difference	t	p
Wireless earbuds ($150 vs. $100)	5.21 (81)	4.73 (85)	$[-0.758, -0.203]$	−3.42	.001
Running shoes ($200 vs. $100)	5.42 (83)	4.74 (93)	$[-0.992, -0.367]$	−4.30	.000
Robot vacuum ($700 vs. $400)	5.37 (89)	4.96 (89)	$[-0.738, -0.094]$	−2.55	.012
Wireless earbuds ($50 vs. $100)	2.62 (87)	3.05 (86)	$[0.161, 0.690]$	3.18	.002

Notes. Entries report means with sample sizes in parentheses, M (N), using only participants who passed attention checks. The 95% confidence interval is for the mean difference, and the t-tests are two-tailed with unequal variances (Welch's t-tests); both compare Personalized Pricing minus Uniform Pricing.