Product Discoverability in E-Commerce Marketplaces with Seller External Advertising

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Abstract

This paper examines how an e-commerce marketplace should design product discovery tools, which help consumers find products on the platform, accounting for sellers potentially advertising on external channels (e.g., search engines and social media) to attract consumers to the marketplace.

Our paper's central finding is that the platform can profit from *strategically limiting* product discovery because of two distinct mechanisms: (1) a *traffic-expansion* mechanism, where limiting discovery mitigates advertising leakage and encourages sellers to advertise, expanding total platform traffic; (2) a *differentiation* mechanism, where limiting discovery induces some but not all sellers to advertise, creating differentiation in consumer awareness among sellers and thus softening price competition. We also show that the platform's optimal discoverability level changes non-monotonically with different market parameters, such as how substitutable competing products are and how costly or effective external advertising is. The strategic interactions of the platform and sellers further yield counterintuitive welfare consequences: when products are stronger substitutes, despite more intense price competition, sellers' profit can increase while consumer surplus can decrease; and when sellers' external advertising becomes less costly or more effective in attracting consumers, both sellers and consumers can be worse off.

Keywords: product discoverability, seller external advertising, e-commerce platform, platform design, recommendation system

INTRODUCTION

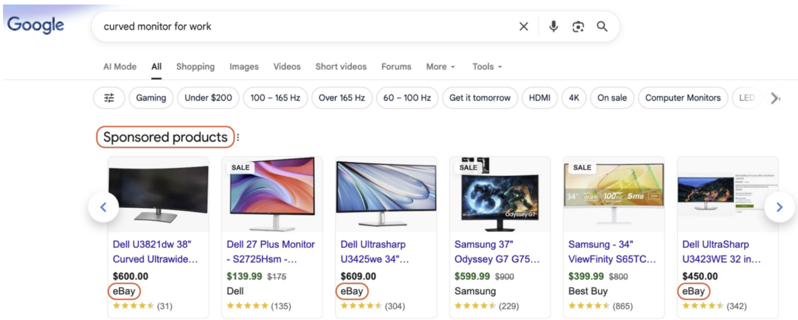
Consumers who shop on e-commerce marketplaces frequently begin their journey outside these marketplaces before visiting them to make a purchase. For example, many shoppers initiate product search via search engines like Google, or encounter products on social media and creator platforms like Instagram and YouTube (eMarketer, 2024; IZEA, 2024; Bertovich, 2025).[1] These external channels contribute significant traffic to e-commerce marketplaces, accounting for roughly 35–50% of total traffic on platforms such as Amazon, Wayfair, and Etsy (Korganbekova and Zuber, 2023; Statista, 2024; Similarweb, 2026).[2] Notably, much of this external traffic is driven by independent marketplace sellers who advertise across various external venues to direct interested consumers to a marketplace. We refer to this practice as *seller external advertising*. Figure 1 presents several examples, such as paid search ads on Google, sponsored posts on Instagram, and influencer marketing on YouTube made by independent sellers. Indeed, seller external advertising has become very prevalent. It is reported that around 40% of Amazon merchants advertised their products on social media or search engines (JungleScout, 2024).

Sellers who run external advertising seek to attract more consumers to purchase their products. However, platforms commonly offer product discovery tools, such as "compare with similar items" recommendations, which lead these consumers to competing products, harming sellers' advertising returns and incentives. Figure 2 presents examples of such tools.[3] Although these tools can help consumers find additional options with potentially better matches and boost their purchase likelihood (Wan, Kumar, and Li, 2024), they can

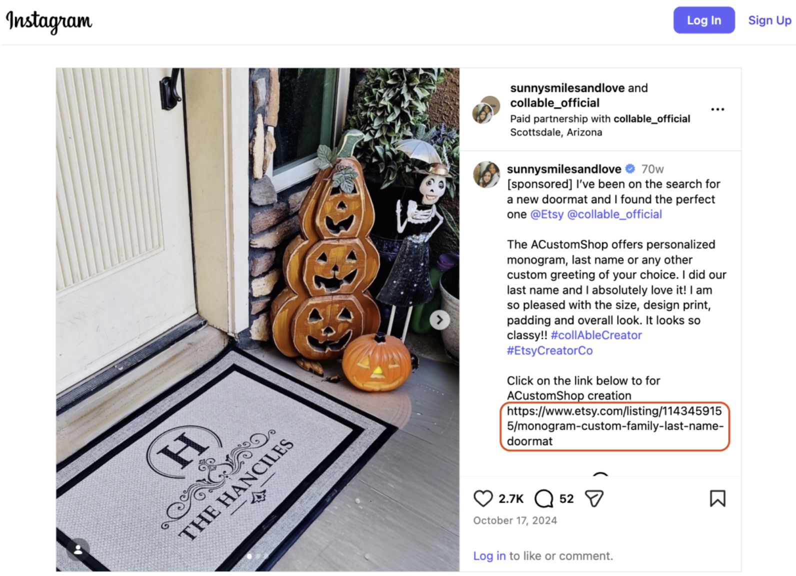
[1] More recently, external AI shopping assistants such as ChatGPT and Perplexity have emerged as another rapidly growing off-platform starting point (Adobe Analytics, 2025; Similarweb, 2025).

[2] Similar patterns hold in other markets, such as Central & Eastern Europe, Brazil, and Japan (Statista, 2025a,b,c).

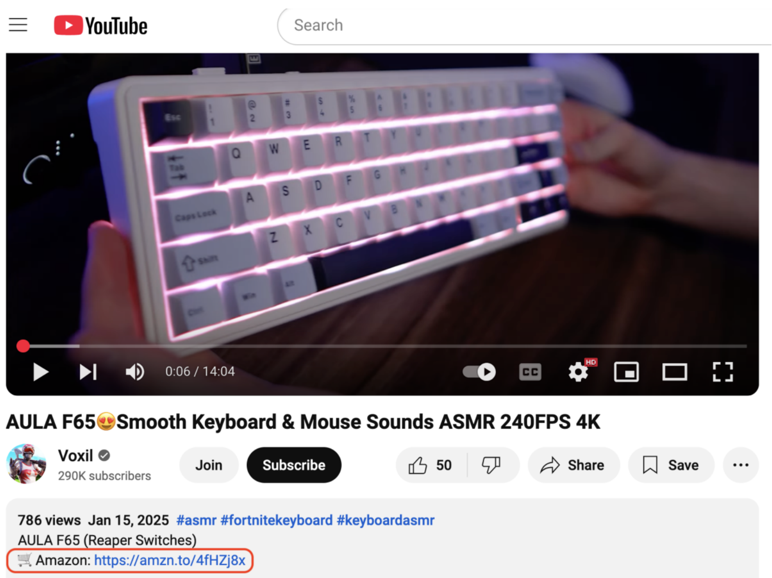
[3] Our paper focuses on discovery tools that recommend alternatives in the same product category, which are commonly used on e-commerce platforms. In practice, there are other formats of product discovery, such as "often bought together" recommendations, that introduce products from other categories. They are beyond the scope of this research.



(a) External ads on Google.com by eBay sellers



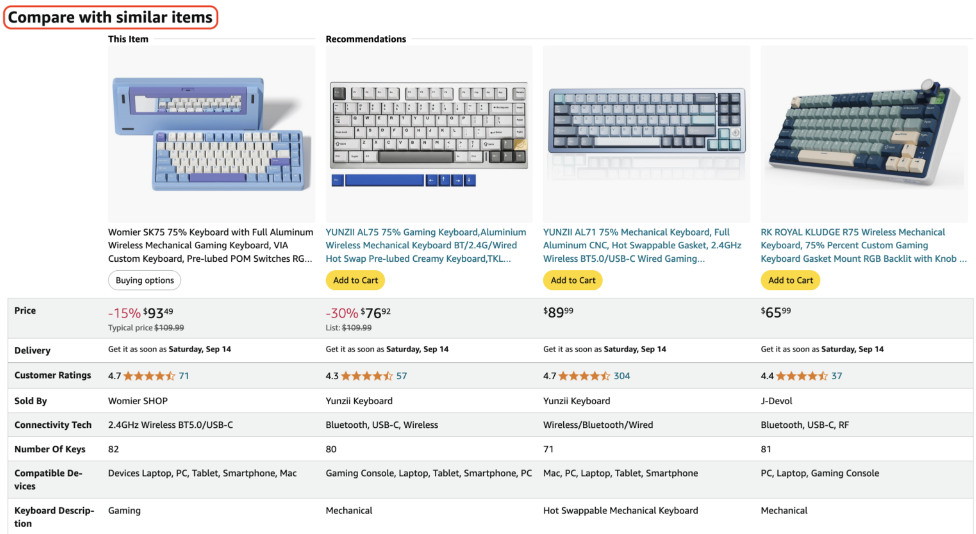
(b) External ad on Instagram by Etsy seller



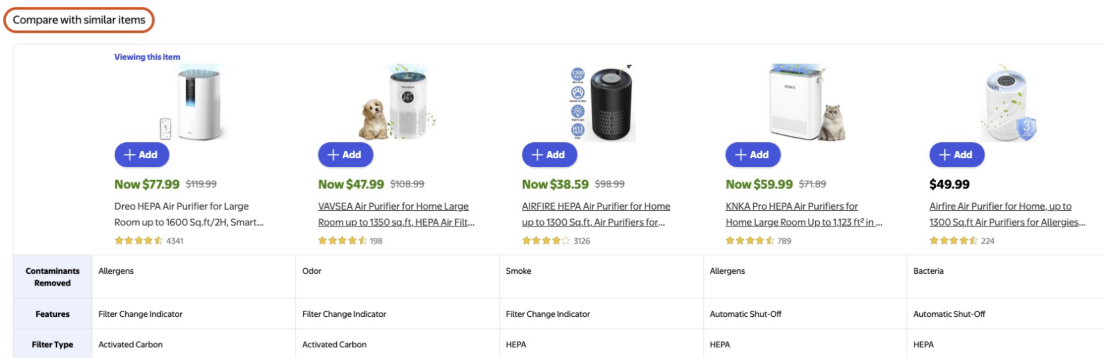
(c) External ad on YouTube by Amazon seller

Figure 1: Examples of seller external advertising on different channels

also lead to *advertising leakage* for sellers: consumers attracted by one seller's external ads end up discovering and buying a competitor's product instead. Such advertising leakage is prevalent and makes sellers hesitant to invest in external advertising. Sellers on Amazon have complained that they "essentially paid for customers to buy somebody else's products" (SPS Commerce, 2022). Various industry guides also caution that Amazon often shows competitors' products to the consumers attracted by a seller's external advertising (e.g., LandingCube, 2025; Click Fluency, 2026; Evolve Media, 2026).



(a) "Compare with similar items" on Amazon



(b) "Compare with similar items" on Walmart

Figure 2: Examples of product discovery recommendations

When sellers invest more in external advertising, a marketplace in turn benefits from the increased consumer traffic. Many platforms, including Amazon, eBay, Walmart, Etsy, and Shopee, have therefore launched active programs and tools to encourage and support sellers' external advertising efforts (eBay Seller Center, 2026; Walmart Marketplace, 2025; Etsy Seller Handbook, 2023; BigSeller, 2025). For instance, Amazon's Brand Referral Bonus program rewards sellers with about 10% of qualifying sales when they drive traffic from external sources, and also provides them with analytics tools and guides (Amazon, 2023a,b). The traffic benefit for marketplaces makes seller external advertising strategically distinct from on-site sponsored ads, which have been the primary focus of the existing literature on e-commerce advertising (e.g., Choi and Mela, 2019; Long, Jerath, and Sarvary, 2022; Long and Liu, 2024). In particular, on-site ads enable a platform to extract surplus from sellers competing for consumers *already browsing on the platform.* By contrast, external ads expand a platform's traffic by *attracting consumers who would not otherwise visit the platform.* To our knowledge, this traffic-acquisition role has received little attention in the literature.

Our study incorporates sellers' strategic external advertising into the analysis of a marketplace's product discovery design. Specifically, we examine the following research questions:

(1) How much product discoverability should the platform provide, accounting for its impacts on sellers' external advertising and consumers' purchase decisions?

(2) How should the platform adjust its product discoverability in response to different market conditions, such as when competing products are stronger (vs. weaker) substitutes, or when external advertising delivers higher (vs. lower) returns?

(3) How does the platform's strategic design of product discoverability affect the equilibrium payoffs of the platform and competing sellers, and the welfare of consumers?

To answer these research questions, we develop a game-theoretic model in which two sellers offer two horizontally differentiated but substitutable products on a platform that charges sellers a percentage commission. The platform chooses a product discoverability level that determines the probability that a consumer who initially knows only one product discovers the competing product. Each seller decides whether to invest in external advertising to attract more consumers to examine its product on the platform, some of whom would not visit the platform otherwise. Consumers evaluate the product(s) they are aware of and make purchase decisions: a consumer who knows only one product decides whether to buy it based on product match and price, whereas a consumer who knows both products decides which one (or neither) to buy.

In a benchmark where sellers do not advertise externally and thus the platform traffic (the number of consumers visiting the platform) is fixed, we find that the platform always benefits from maximizing discoverability. This is because doing so maximizes consumers' purchase likelihood. In sharp contrast, accounting for sellers' strategic decisions on external advertising, we demonstrate that the platform can benefit from limiting product discovery, because doing so incentivizes sellers' external advertising by reducing advertising leakage. This insight echoes Amazon's response to the aforementioned sellers' complaints about advertising leakage: the platform toned down prominent competitor recommendations and promised that external ads "push traffic directly to [sellers'] product pages with zero competitors at the top of your listing" (SPS Commerce, 2022).

Moreover, our analysis reveals two distinct mechanisms through which limiting discovery benefits the platform. The first is a *traffic-expansion* mechanism. When external advertising primarily reaches new consumers who would not otherwise visit the platform, it can substantially expand platform traffic. By limiting discovery, the platform mitigates advertising leakage and encourages sellers to advertise externally, expanding total platform traffic and generating more commissions, even though reduced discovery lowers each visitor's purchase likelihood. The second is a *differentiation* mechanism. Even in the absence of the traffic-expansion mechanism (for example, when external advertising only informs existing platform visitors of additional sellers without bringing new consumers), limiting discovery can induce asymmetric advertising across sellers: some sellers invest in external advertising whereas others do not. This asymmetry creates differentiation in consumer awareness and softens price competition, raising the platform's commission revenue.

Interestingly, we find that the platform's optimal product discovery design can vary non-monotonically with product substitutability (the chance a consumer matched with one product is also matched with the other) and advertising cost-effectiveness (the number of platform visits generated per dollar of external ad spending). When products are weak substitutes, advertising leakage due to product discovery is limited, so the platform can maintain high discovery to boost consumers' purchase likelihood while still inducing sellers to advertise. When products are moderate substitutes, product discovery leads to considerable leakage and thus significantly discourages sellers' external advertising. In response, the platform limits discovery to ensure sellers advertise. When products are strong substitutes, incentivizing advertising would require limiting discovery so much that the loss in purchase likelihood is too costly for the platform, so the platform restores high discovery although sellers do not advertise. Finally, when products are very strong substitutes, the benefit from boosting purchase likelihood diminishes because a consumer mismatched with one product is likely mismatched with the other as well; in this region, the platform again limits product discovery to curb leakage. Turning to the cost-effectiveness of external advertising, when it is very high, sellers advertise regardless, so the platform provides easy discovery. When it is very low, encouraging advertising would require restricting discovery too much. Therefore, only with moderate cost-effectiveness does the platform profitably limit product discovery to encourage external advertising. These results imply that platforms should customize product discovery design for different product categories based on their market characteristics.

The platform's strategic discoverability design and the resulting sellers' strategic advertising responses can lead to counterintuitive welfare implications for market players. First, when sellers' products are stronger substitutes, despite fiercer price competition, sellers' profit can increase whereas consumer surplus can decrease. The former happens when stronger substitutability leads the platform to make product discovery easier, which discourages sellers from advertising and, in equilibrium, saves their advertising costs. The latter occurs when stronger substitutability leads the platform to limit product discovery, which reduces consumers' chances of finding a matching product. Second, when sellers have higher cost-effectiveness in external advertising, they can become worse off, and consumer surplus can decrease. This occurs when higher cost-effectiveness induces the platform to limit discovery to encourage advertising. For sellers, the benefit from expanded traffic is insufficient to cover the advertising expense. For consumers, the limited product discovery hurts their chances of finding a matching product. In addition, we find that when external advertising becomes more costly, the platform can benefit more from the differentiation mechanism and receive a higher profit.

Lastly, we demonstrate the robustness of our qualitative results in several alternative model settings. In particular, our key insights regarding the impacts of product discoverability continue to hold even when the platform endogenously determines the commission rate. In this extended setting, the platform faces an additional trade-off: a high commission rate improves its expected profit from each visiting consumer but discourages sellers' external advertising investment. We show that the platform can profitably boost sellers' external advertising by reducing both the discoverability level and the commission rate.

On the theoretical front, we contribute a model that considers sellers' strategic external advertising in analyzing a marketplace's product discovery design. We reveal the distinct role of external advertising in attracting new traffic to a platform, contributing to the existing platform literature that has focused on sponsored on-site ads. We identify two mechanisms, traffic-expansion and differentiation, through which limiting product discovery benefits the platform. On the managerial front, our findings show how platforms should tailor product discovery based on how substitutable competing products are, how cost-effective sellers' external advertising is, and whether external advertising primarily expands traffic or reaches existing visitors. We also caution that the platform's strategic design of product discovery can generate unexpected welfare consequences.

LITERATURE

Our paper contributes to the broad literature on platform design. Existing studies have examined optimal designs of platform features such as rating and reputation systems (Hui et al., 2016; Shi, Srinivasan, and Zhang, 2023; Ke, Sun, and Jiang, 2024), platform access (Halaburda, Jan Piskorski, and Yıldırım, 2018; Huang, Lyu, and Xu, 2022; Teh, 2022), revenue structures (Choi and Mela, 2019; Lin, 2020; Long, Jerath, and Sarvary, 2022), and data and information sharing (Liu, Zhang, and Zhang, 2021; Liu and Long, 2025). We focus on a platform's design of product discovery features and how it shapes sellers' incentives to advertise externally.

Our paper is thus closely related to the literature on product discovery on platforms. An important research question in this literature is how platforms can influence vendor competition by strategically designing their product discovery environments, such as search (Baye and Morgan, 2001; Gu and Liu, 2013; Dukes and Liu, 2016; Jiang and Zou, 2020; Zhong, 2023; Zou and Zhou, 2025; Ke, Lin, and Lu, 2026), and product recommendations (Li, Chen, and Raghunathan, 2018, 2020; Teh and Wright, 2022; Zhou and Zou, 2023; Hagiu and Wright, 2024). For example, Dukes and Liu (2016) find that a platform should set consumers' search cost at an intermediate level to prevent consumers from searching too many products while still ensuring that they investigate each item in sufficient depth. Hagiu and Wright (2024) find that a platform can reduce product discoverability to alleviate price competition between vendors. In the offline retailing context, Gu and Liu (2013) find that a retailer can strengthen its channel power by displaying competing products in distant locations, a strategy analogous to reducing discoverability, which incentivizes manufacturers to compete for more prominent placement.

These studies, whether in the online or offline context, take consumer traffic to a platform (or retailer) as given, whereas our research considers platform traffic as an endogenous outcome from the strategic interaction between the platform and its sellers. In our setting, the platform's design of product discoverability not only affects sellers' pricing competition as the literature suggests, but also affects sellers' external advertising incentives. We identify two distinct mechanisms through which restricting discoverability benefits the platform: mitigating advertising leakage to preserve sellers' returns from traffic acquisition, and inducing seller differentiation to soften price competition. We also show when the platform should restrict discoverability in light of these two mechanisms, along with unexpected welfare consequences.

Our study models a seller's external advertising as a means of generating consumer awareness of its product and thus is related to the literature on awareness advertising. Empirical evidence confirms that advertising can significantly increase brand awareness (Clark, Doraszelski, and Draganska, 2009). Analytical work in this area examines how awareness advertising shapes price competition: it can intensify competition (Grossman and Shapiro, 1984), but may also lead to higher prices when product differentiation is high or consumers' product valuation is low (Soberman, 2004; Amaldoss and He, 2010). Other studies also examine targeted awareness advertising (Iyer, Soberman, and Villas-

Boas, 2005) and firms' joint choice of quality and advertising (Lauga, Ofek, and Katona, 2022). Some recent studies consider sellers' on-site advertising (e.g., sponsored ads) on e-commerce marketplaces (Choi and Mela, 2019; Long, Jerath, and Sarvary, 2022; Long and Liu, 2024; Long and Amaldoss, 2024; Saremi and Subramanian, 2026). In contrast, we focus on sellers' advertising through external channels. Distinct from on-site advertising, which helps sellers compete for shoppers already browsing on a platform, sellers' off-platform advertising plays a top-of-the-funnel role by bringing additional shoppers to the platform in the first place.

MODEL

Overview

We consider two sellers $j \in \{A, B\}$ selling two competing products on a platform, which charges a percentage commission. For convenience, we use $-j$ to denote the competitor of j; for example, $-j = B$ if $j = A$. The platform chooses a product discoverability level $x \in [0, 1]$ that governs the probability that a visiting consumer who initially knows only one product discovers the competing product. In particular, for visitors who know only product j, a fraction x discover the other product $-j$ and so know both, whereas the remaining fraction $1 - x$ still know only product j. Discoverability does not affect visitors to the platform who already know both products.

Each seller sets a product price and decides whether to invest in external advertising to raise consumer awareness of its product. Consumers make purchase decisions based on product awareness, product match, and product prices. Below, we describe the specifics of the consumers, the two sellers, and the platform.

Consumers

Consumers have a unit demand and are heterogeneous in two dimensions: (1) their preferences for the two products, which depend on product match, and (2) their awareness of the products, which depends on sellers' external advertising. We describe them in order.

Consumers' product preferences

The two products are horizontally differentiated and imperfectly substitutable. Each product matches a fraction m of consumers, yielding value $v > 0$ to a matched consumer and zero to an unmatched one. A fraction αm of consumers match both products, where α measures the *substitutability* between products: a higher α means a consumer who finds one product matching (mismatching) is more likely to find the other product also matching (mismatching). Table 1 summarizes consumer composition by product match. We assume $0 < m \leq \frac{1}{2-\alpha}$ to ensure model regularity, so that the "match neither" group has non-negative size. We also restrict attention to $0 < \alpha < 1/2$ throughout the paper; the case $\alpha \geq 1/2$ yields the trivial outcome in which seller price competition is so intense that the platform always minimizes product discoverability (i.e., $x = 0$) to soften competition.

Table 1: Consumer composition by product match

	Match both A and B	Match only A	Match only B	Match neither
Fraction	αm	$(1-\alpha)m$	$(1-\alpha)m$	$1-m(2-\alpha)$

A consumer's utility from buying product j is $u_j = v - p_j$ if the product provides a match and $u_j = -p_j$ otherwise, where p_j is the product price. If the consumer exits the platform without purchasing, we normalize the utility from the outside option to zero. Without loss of generality, we assume $0 < p_j \leq v$ in this paper.

Consumers' product awareness

Consumers may or may not know a product, and a seller's external advertising raises the mass of consumers who know it. We use $I_j \in \{Y, N\}$ to indicate whether seller j advertises externally. For tractability, our main model treats the external advertising decision as binary, assumes that advertising only increases awareness, and assumes that when both sellers advertise, their ads reach disjoint sets of consumers. In Section "Extensions," we show that our key insights continue to hold qualitatively (a) when advertising levels are continuous, (b) when advertising can also be persuasive, increasing consumers' product valuations, and (c) when consumers may encounter both sellers' ads.

We assume that when neither seller advertises (i.e., $I_A = I_B = N$), a unit mass of consumers know product A, another unit mass know product B, and the rest know neither product. Our key insights remain qualitatively unchanged when some consumers know both products without advertising.

When seller j advertises externally ($I_j = Y$), a mass d of consumers previously unaware of product j are now aware of it. Moreover, we assume that among these consumers reached by seller j's ad, a fraction $\lambda \in [0, 1]$ initially know neither product and, after receiving seller j's ad, become aware of product j only. The remaining fraction $1 - \lambda$ already know the competing product $-j$ and, after receiving seller j's ad, become aware of both products. We assume $0 < d < \frac{1}{1-\lambda}$ such that the size of the second group is smaller than one. Parameter λ can reflect market maturity. In emerging product categories, many consumers have not yet heard of any product, so external advertising tends to bring new consumers to the platform (i.e., a large λ). In mature, saturated markets, most consumers already know at least one product, so external advertising tends to reach consumers who already know the competing product (i.e., a small λ).

We assume that consumers will visit the platform *if and only if* they know at least one product, and these consumers constitute the platform's traffic. In Section "Extensions," we show that our results are qualitatively robust when the platform also has visiting consumers who do not know any product.

Let s_j denote the mass of consumers who know only product j before visiting the platform, and s_{AB} the mass of consumers who know both products before visiting the platform. Table 2, panel (a) summarizes how sellers' external advertising efforts affect awareness composition among consumers *before visiting the platform or any platform discovery.* The first three columns of panel (a) report the composition of consumer awareness; the last column shows the total platform traffic, which increases when more sellers advertise and when advertising reaches more consumers who initially know neither product (i.e., a larger λ).

Table 2, panel (b) summarizes the resulting awareness composition *after visiting the platform and potentially discovering new products.* As expected, panel (a) coincides with the $x = 0$ case in panel (b). Compared with panel (a), platform discovery shifts consumers from knowing only one product (first two columns) to knowing both (third column), whereas the total traffic remains unchanged.

Table 2: Consumer awareness composition

(a) Before visiting the platform

Seller advertising	Know only A (s_A)	Know only B (s_B)	Know both (s_{AB})	Total traffic ($s_A + s_B + s_{AB}$)
$I_A = I_B = N$	1	1	0	2
$I_A = Y, I_B = N$	$1 + \lambda d$	$1 - (1 - \lambda)d$	$(1 - \lambda)d$	$2 + \lambda d$
$I_A = N, I_B = Y$	$1 - (1 - \lambda)d$	$1 + \lambda d$	$(1 - \lambda)d$	$2 + \lambda d$
$I_A = I_B = Y$	$1 + \lambda d - (1 - \lambda)d$	$1 + \lambda d - (1 - \lambda)d$	$2(1 - \lambda)d$	$2 + 2\lambda d$

(b) After product discovery on the platform

Seller advertising	Know only A	Know only B	Know both	Total traffic
$I_A = I_B = N$	$(1 - x)$	$(1 - x)$	$2x$	2
$I_A = Y, I_B = N$	$(1 - x)(1 + \lambda d)$	$(1 - x)[1 - (1 - \lambda)d]$	$(1 - \lambda)d + x[2 + (2\lambda - 1)d]$	$2 + \lambda d$
$I_A = N, I_B = Y$	$(1 - x)[1 - (1 - \lambda)d]$	$(1 - x)(1 + \lambda d)$	$(1 - \lambda)d + x[2 + (2\lambda - 1)d]$	$2 + \lambda d$
$I_A = I_B = Y$	$(1 - x)[1 + (2\lambda - 1)d]$	$(1 - x)[1 + (2\lambda - 1)d]$	$2\{(1 - \lambda)d + x[1 + (2\lambda - 1)d]\}$	$2 + 2\lambda d$

Sellers

Each seller $j \in \{A, B\}$ decides two strategic variables: its external advertising decision I_j and product price p_j. If a seller advertises externally ($I_j = Y$), it incurs a fixed cost $k > 0$ and expands its product awareness by d, as described above. If a seller does not advertise ($I_j = N$), it incurs no cost and retains only its baseline awareness of one. A higher d or a lower k indicates higher cost-effectiveness of external advertising. For example, recent advances in AI-based analytics and programmatic ad buying have enhanced the cost-effectiveness of sellers' external advertising campaigns.[4] In practice, k/d can be interpreted as the pay-per-click cost of attracting a mass of consumers to visit the platform via an ad.

Each seller maximizes its selling profit net of advertising cost, given by

$$\Pi_j = (1 - r) \cdot p_j \cdot D_j - k \cdot \mathbb{1}[I_j = Y], \tag{1}$$

where D_j is seller j's demand, and $r \in (0, 1)$ is the platform's commission rate. We normalize sellers' marginal production costs to zero.

Platform

The platform's key choice variable is the product discoverability level $x \in [0, 1]$. In practice, higher discoverability means that the platform places more discovery features, such as product comparison tools and "compare with similar items" recommendations, on a product's webpage. Our model provides a stylized representation of how product discovery features expand consumers' consideration sets. This modeling approach allows us to focus on how product discoverability influences sellers' advertising and pricing decisions, and

[4]Third-party tools such as Carbon6's PixelMe (Carbon6, 2025) and Ampd's Flywheel (Ampd, 2025) can support external advertising efforts via campaign automation and attribution.

therefore the platform's incentives to adjust discoverability.[5] Our approach is in a similar spirit to Long and Amaldoss (2024), which captures how sponsored advertising on a platform increases the probability that a product is included in consumers' consideration sets. We assume zero cost for implementing product discovery, which allows us to show that the platform may strategically restrict discoverability even without any cost consideration.

The platform maximizes total commission revenue $r \cdot (p_A \cdot D_A + p_B \cdot D_B)$. In the main model, we fix the commission rate $r \in (0, 1)$ to focus on the platform's choice of discoverability x. In Section "Extensions," we investigate the case in which the platform endogenously sets both r and x.

Game sequence. The game unfolds as follows. In Stage 1, the platform chooses the product discoverability level. In Stage 2, the two sellers simultaneously decide whether to invest in external advertising. In Stage 3, the two sellers simultaneously set product prices. Lastly, in Stage 4, consumers visit the platform and make purchase decisions. The assumption that sellers make advertising decisions before pricing is common because awareness advertising typically requires more planning and lead time to become effective, involving budgeting, producing creatives, and waiting for consumer exposure to the ads (Ehrenberg, 1959; Assmus, Farley, and Lehmann, 1984). By contrast, sellers can adjust product prices flexibly as awareness changes.

ANALYSIS

We solve the game using backward induction. The Web Appendix collects all proofs and details, including the definitions for all thresholds (e.g., x_1 and x_2). Tables and figures numbered with the prefix 'W' (e.g., Table W1) are also provided there.

[5] An interpretation of our setting is that consumers have heterogeneous search costs for finding products beyond the one they initially know. A fraction x of consumers have negligible search costs and thus can find both products, whereas the remaining fraction $1 - x$ have prohibitively high search costs and thus consider only the focal product. Higher discoverability reduces consumers' search costs, shifting more consumers to the former group.

Table 3: Summary of key model notation

Symbol	Meaning
$x \in [0, 1]$	Product discoverability level
r	Percentage commission rate charged by the platform
p_j	Price of product $j \in \{A, B\}$
$I_j \in \{Y, N\}$	External advertising decision of seller j: Y—advertise; N—not advertise
s_j, s_{AB}	Consumer awareness before visiting platform: s_j—know only j; s_{AB}—know both
d	Effectiveness of seller external advertising
k	Cost of seller external advertising
λ	Extent to which seller external advertising expands platform traffic
m	Match probability of a product (overall appeal of product category)
α	Substitutability between products A and B
v	Value of a matched product
u_j	Consumer utility of buying product j
Π_j, Π_P	Seller j's profit and the platform profit
n_j	Mass of "j-loyals", consumers who may buy only product j
n_{AB}	Mass of "switchers", consumers who may buy either product

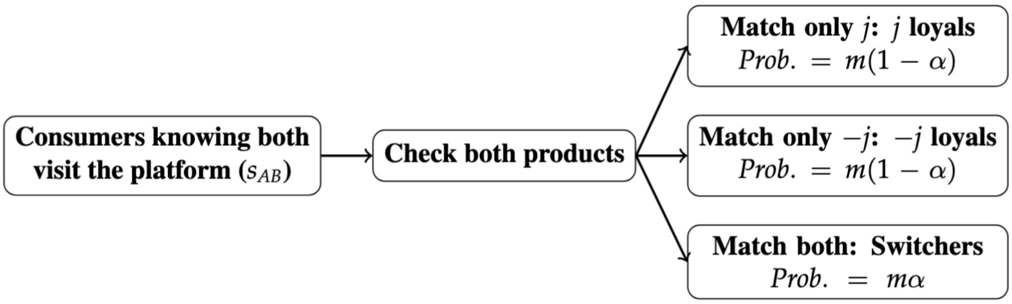
Consumer Shopping Journey and Demand Composition

In Stage 4, consumers' purchase decisions depend on product discovery, product match, and product prices. Figure 3 illustrates the shopping journey for each consumer type.

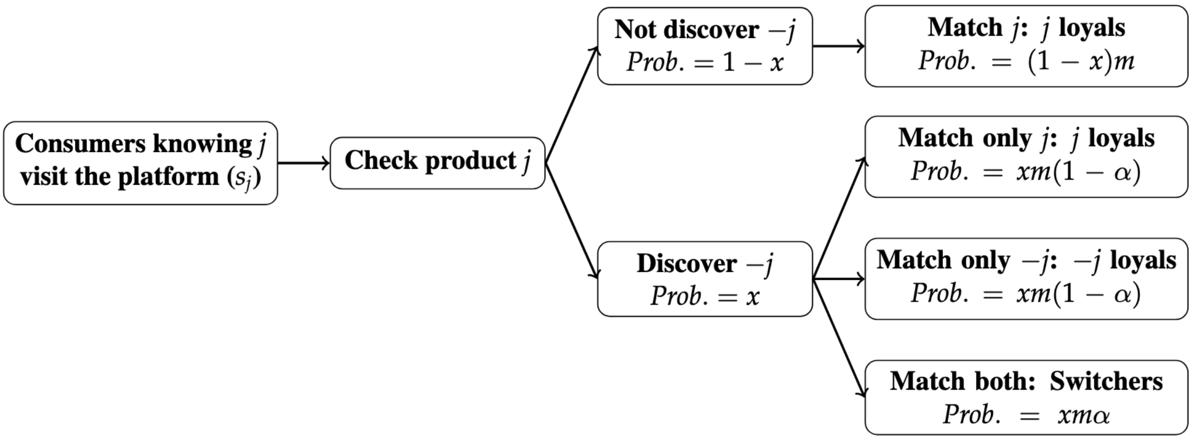
Consumers who know both products before visiting the platform (see Figure 3a): Upon visiting the platform, these consumers evaluate both products. If both products match, they buy the lower-priced one (choosing randomly when prices are equal). If only one product matches, they buy that product. If neither matches, they leave without purchasing.

Consumers who know only product $j \in \{A, B\}$ before visiting the platform (see Figure 3b): Upon visiting the platform, these consumers evaluate product j and, with probability x, also discover product $-j$. If they do not discover product $-j$, they buy product j when it matches and otherwise leave without purchasing. If they discover product $-j$, the four purchase scenarios mirror those in the first case (see Figure 3a).

Three segments of purchasing consumers arise on the platform: "A-loyals," who are



(a) When consumers know both products before visiting the platform



(b) When consumers know only one product before visiting the platform

Figure 3: Consumer shopping journey

aware of and matched with product A only; "B-loyals," who are aware of and matched with product B only; and "switchers," who are aware of and matched with both products and buy the lower-priced one. This demand composition parallels Varian (1980) and Narasimhan (1988). We adopt the terms "loyals" and "switchers" to align with the extant literature; they are not intended to imply literal brand loyalty in our setting. We let n_A, n_B, and n_{AB} denote the mass of A-loyals, B-loyals, and switchers, respectively. These are given by:

$$n_A = s_A(1-x)m + [(s_A+s_B)x + s_{AB}]m(1-\alpha); \tag{2}$$

$$n_B = s_B(1-x)m + [(s_A+s_B)x + s_{AB}]m(1-\alpha); \tag{3}$$

$$n_{AB} = [(s_A+s_B)x + s_{AB}]m\alpha. \tag{4}$$

As shown in panel (a) of Table 2, the values of s_A, s_B, and s_{AB} in equations (2)–(4) depend on sellers' external advertising decisions. Table W1 in the Web Appendix summarizes the demand composition under each advertising scenario.

Given $0 < p_j \leq v$, A-loyal consumers always buy product A, B-loyal consumers always buy product B, and switchers buy the lower-priced product. Consumer demand for product $j \in \{A, B\}$ as a function of prices is therefore:

$$D_j = \begin{cases} n_j + n_{AB} & \text{if } p_j < p_{-j}, \\ n_j + \frac{1}{2}n_{AB} & \text{if } p_j = p_{-j}, \\ n_j & \text{if } p_j > p_{-j}. \end{cases} \tag{5}$$

The total demand on the platform is given by $D_P = D_A + D_B = n_A + n_B + n_{AB} = (s_A + s_B)(1 + x - \alpha x)m + s_{AB}(2-\alpha)m$.

Sellers' Pricing and External Advertising Decisions

In Stage 3, the two sellers simultaneously set product prices to maximize their profit Π_j, as defined in equation (1), taking as given their advertising decisions and the platform's product discoverability choice. Following Narasimhan (1988), the pricing subgame has a unique mixed-strategy equilibrium. Table W2 in the Web Appendix summarizes sellers' expected equilibrium profits in Stage 3.

In Stage 2, the two sellers simultaneously decide whether to invest in external advertising, anticipating the pricing subgame in Stage 3 and taking product discoverability on the platform as given. We summarize sellers' equilibrium decisions on external advertising in the following lemma. Throughout the paper, by higher "cost-effectiveness" of external advertising, we mean a higher d or a lower k.[6]

Lemma 1 (Equilibrium seller external advertising)**.** *In equilibrium:*

(i) Neither seller invests in external advertising if it has low cost-effectiveness, i.e., $d \leq d_1$ *(or equivalently,* $k \geq k_2$*).*

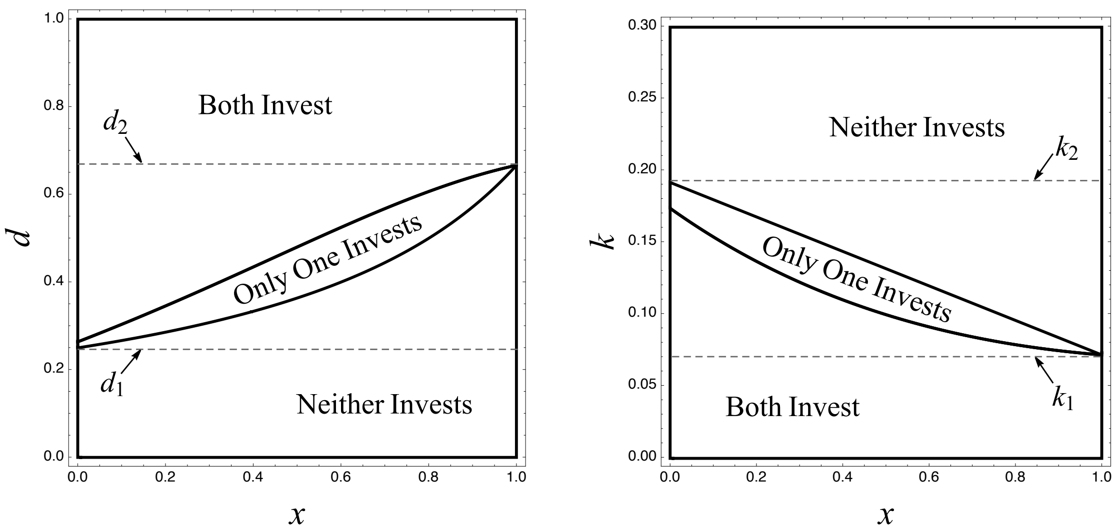
(ii) If external advertising has moderate cost-effectiveness, i.e., $d_1 < d < d_2$ *(or equivalently,* $k_1 < k < k_2$*), higher product discoverability* x *reduces the number of sellers that invest in external advertising. Specifically, there exist thresholds* $0 \leq x_1 < x_2 < 1$ *such that both sellers invest when* $x < x_1$*, only one seller invests when* $x_1 < x < x_2$*, and neither seller invests when* $x > x_2$*.*

(iii) Both sellers invest in external advertising if it has high cost-effectiveness, i.e., $d \geq d_2$ *(or equivalently,* $k \leq k_1$*).*

Lemma 1 characterizes how the platform's product discoverability interacts with the cost-effectiveness of external advertising to shape sellers' equilibrium advertising decisions. Figure 4 illustrates these results. A seller's incentive to advertise depends on whether its net benefit from advertising covers its cost k, i.e., whether $\Delta\Pi \equiv \Pi_j|_{I_j=Y,I_{-j}} - \Pi_j|_{I_j=N,I_{-j}} \geq 0$.[7] We can show that higher product discoverability x weakens sellers' external advertising incentives ($\frac{\partial \Delta\Pi}{\partial x} < 0$), because higher x leads to greater leakage from external advertising:

[6] We keep both parameters because the equilibrium outcomes cannot be summarized by a single cost-effectiveness index such as d/k or $d - k$.

[7] The expression for $\Delta\Pi$ depends on the competitor's advertising decision, I_{-j}. That said, all subsequent comparative statics regarding $\Delta\Pi$ remain valid regardless of whether the competitor advertises. For expositional simplicity, we therefore use the notation $\Delta\Pi$ without specifying I_{-j} throughout the paper.



(a) Effects of advertising effectiveness d and product discoverability x

(b) Effects of advertising cost k and product discoverability x

Figure 4: Equilibrium outcomes of seller external advertising

Note. $v = 2$, $r = 0.5$, $m = 0.4$, $\alpha = 0.4$, and $\lambda = 0.5$. $k = 0.08$ in (a) and $d = 0.6$ in (b).

consumers attracted by a seller's advertising are more likely to discover and potentially buy the competitor's product on the platform.[8] This dampening effect, however, is effective only when external advertising has moderate cost-effectiveness. When cost-effectiveness is very high, both sellers advertise despite the leakage; when cost-effectiveness is very low, neither seller advertises even in the absence of leakage.

Moreover, a seller's incentive to advertise depends on whether its rival advertises. If the rival already advertises, the focal seller, by choosing not to advertise, can benefit from reduced price competition arising from the asymmetry in their loyal segment sizes. This gives rise to asymmetric equilibria when the platform implements a moderate level of discoverability (i.e., $x_1 < x < x_2$): only one seller advertises in equilibrium despite the two sellers' symmetric product offerings. The non-advertising seller free rides on the consumer traffic brought by its rival, since some of these additional consumers may discover the non-advertiser's product on the platform.

[8]Moreover, the net benefit $\Delta\Pi$ increases with advertising effectiveness d, traffic-expansion extent λ, and product match probability m, whereas it decreases with advertising cost k and product substitutability α.

Lemma 1 has an important implication for the platform's discoverability decision: higher product discoverability increases visiting consumers' purchase likelihood but also tends to reduce overall platform traffic by inducing advertising leakage. As discussed in the Introduction, this leakage issue is practically significant and well documented by abundant experimental and anecdotal evidence (Kumar and Hosanagar, 2019; SPS Commerce, 2022; LandingCube, 2025). Next, we examine the platform's optimal choice of product discoverability in light of this trade-off.

Platform's Product Discoverability Decision

In Stage 1, the platform chooses the level of product discoverability $x \in [0, 1]$ to maximize its total commission revenue from product sales, that is, $\Pi_P(x) = r\mathbb{E}[p_A D_A + p_B D_B]$, anticipating the sellers' and consumers' subsequent decisions.

Benchmark case: sellers do not advertise externally. We first consider a benchmark case in which sellers do not advertise externally, so the total consumer traffic to the platform is fixed. In this case, Lemma 2 shows that the platform always sets the highest discoverability level, which maximizes the purchase likelihood of visiting consumers.

Lemma 2. *When sellers do not advertise externally, the platform chooses the highest product discoverability* $x^* = 1$.

Sellers strategically decide on external advertising. This section delivers one of our central messages: *the platform may strategically limit product discoverability to* $x^* < 1$ *when sellers endogenously decide on external advertising.* Moreover, we identify two distinct mechanisms for the platform to limit discoverability. To clearly illustrate the two mechanisms, the main text focuses on two polar cases, $\lambda = 1$ and $\lambda = 0$.

When $\lambda = 1$, a seller's external advertising fully translates into new platform traffic. We use this scenario to demonstrate a *traffic-expansion mechanism*: the platform limits discoverability to encourage external advertising and thereby expand consumer traffic. On the other hand, when $\lambda = 0$, advertising does not expand any platform traffic, so the traffic-expansion mechanism is completely shut down. We use this scenario to show that the platform still has an incentive to limit discoverability due to a *differentiation mechanism*: by inducing some but not all sellers to advertise with an appropriate discoverability level, the platform creates differentiation across sellers that softens price competition and raises commission revenue. At the end of this section, we investigate the general case of $\lambda \in (0, 1)$.

External advertising fully expands platform traffic ($\lambda = 1$***)***

We begin with the case of $\lambda = 1$, where external advertising fully expands platform traffic. As previewed above, the platform may find it optimal to limit product discoverability to incentivize external advertising and bring more consumer traffic. Proposition 1 formally establishes this result, showing that product substitutability has a non-monotonic impact on the optimal discoverability. Proposition 2 further complements this finding by examining how advertising cost-effectiveness affects the platform's discoverability decision.

Proposition 1 (Equilibrium product discoverability with seller external advertising)**.** *Consider $\lambda = 1$. In equilibrium, the platform limits product discoverability to $x^* < 1$ when product substitutability α is moderate or very high; otherwise, the platform chooses the highest discoverability $x^* = 1$.*

Mathematically, $x^* < 1$ when $\alpha \in (\alpha_l, \alpha_m]$ or $\alpha \in (\alpha_h, 1/2)$; $x^* = 1$ when $\alpha \in (0, \alpha_l]$ or $\alpha \in (\alpha_m, \alpha_h]$. Thresholds $\alpha_l \le \alpha_m \le \alpha_h$ are defined in the proof.[9]

For visual illustration of Proposition 1, Figure 5(a, first row) provides an example of how x^* changes with α. Figure 6 provides another illustration of the platform's optimal product discoverability and the sellers' equilibrium advertising decisions. Along the red dashed vertical line in Figure 6, the optimal discoverability shifts three times when α

[9] Some of these intervals, as well as those in other propositions, may be empty depending on the parameters.

increases from 0 to 1/2: first from the highest level of $x^* = 1$ to a constrained level of $x^* < 1$, then back to the highest level, and lastly to a constrained level again.[10]

To understand Proposition 1, we first identify three effects of higher discoverability x:

- *Ad-discouraging effect*: as Lemma 1 shows, a higher x weakens sellers' incentives to invest in external advertising, reducing platform traffic.
- *Competition-intensifying effect*: a higher x leads more consumers to discover both products, raising the proportion of "switchers" relative to "loyals" and intensifying price competition.
- *Match-boosting effect*: a higher x raises consumers' likelihood of finding a matching product, increasing their purchase likelihood.

The first two effects reduce the platform's profit, whereas the third increases its profit.

Next, we explain how product substitutability α moderates the three effects. A higher α means that when a consumer matches with one product, she is also more likely to match with the other upon discovering it. Thus, a higher α *aggravates* the ad-discouraging effect and the competition-intensifying effect. By contrast, a higher α *attenuates* the match-boosting effect. With stronger substitutes, a consumer who mismatches with one product is also likely to mismatch with the other, so discovering an additional product yields a smaller gain in her overall purchase likelihood. Formally, product discovery raises a consumer's purchase probability from m to $(2 - \alpha)m$, a gain of $(1 - \alpha)m$ that shrinks as α increases.

We now explain how the platform's optimal discoverability changes with α across the four regions in Proposition 1. In each region, the effects discussed above shape how much the platform gains from expanded traffic and softer competition by reducing discoverability and whether the resulting loss in purchase likelihood is worth bearing.

[10] In the region of "At Least One Invests", either only one seller invests or both sellers invest in equilibrium. We derive the parameter boundary between these two advertising cases in the Web Appendix.

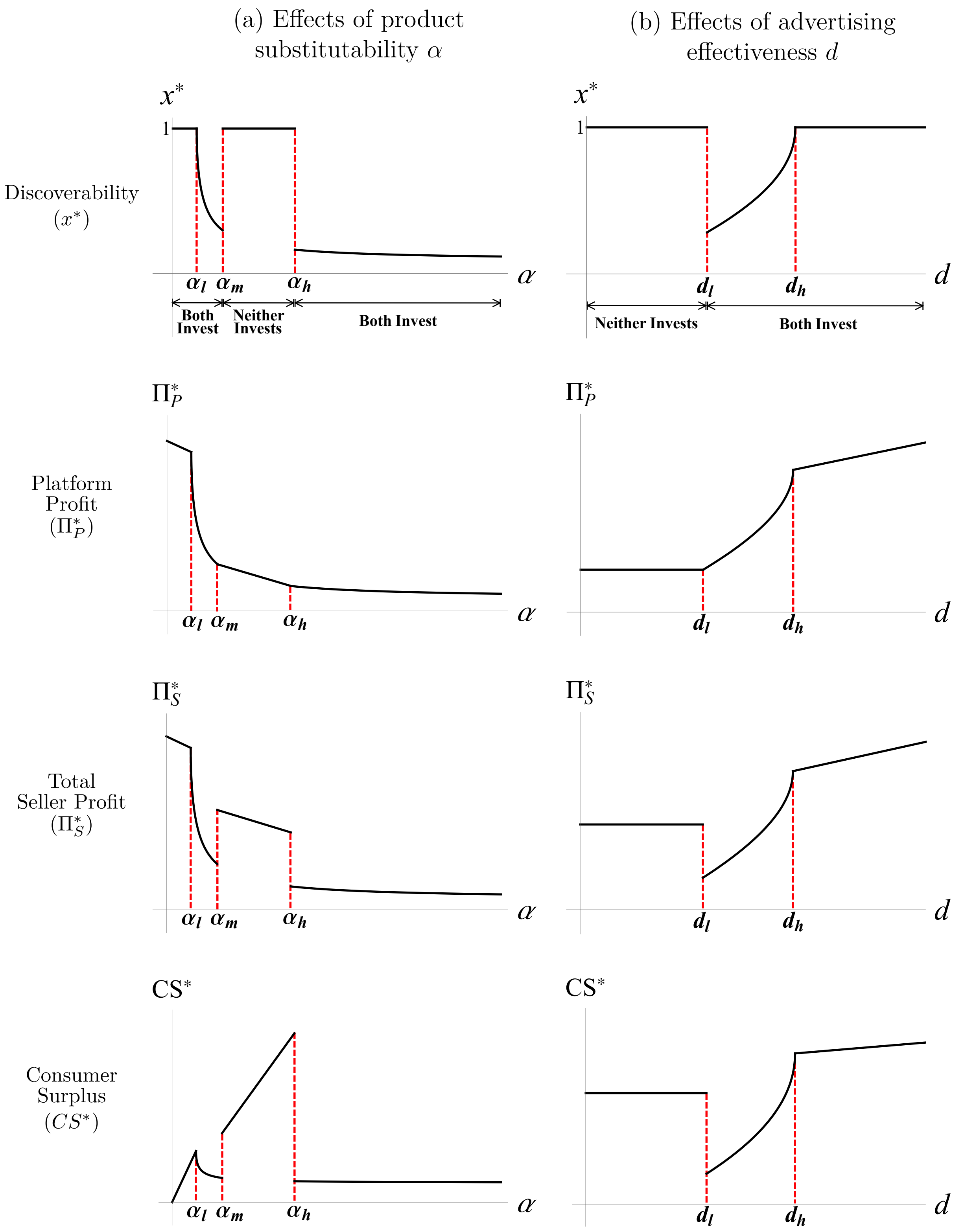
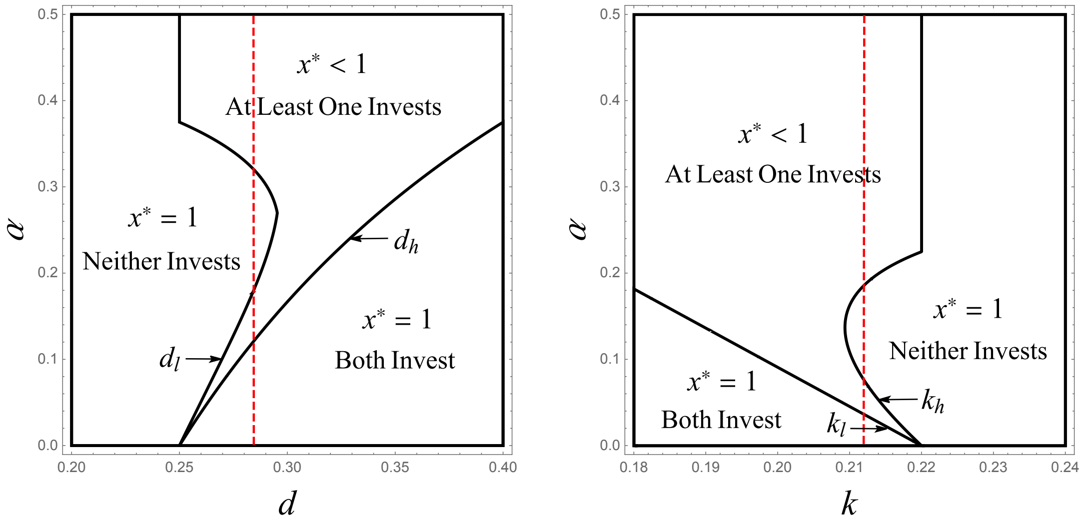


Figure 5: Effects of key parameters on equilibrium outcomes

Note. $v = 2$, $r = 0.5$, $m = 0.4$, and $\lambda = 1$. In (a), $d = 0.55$ and $k = 0.212$; in (b), $k = 0.1$ and $\alpha = 0.3$.



(a) Effects of advertising effectiveness d and product substitutability α

(b) Effects of advertising cost k and product substitutability α

Figure 6: Equilibrium product discoverability and seller external advertising

Note. $v = 2$, $r = 0.5$, $m = 0.4$, and $\lambda = 1$. $k = 0.1$ in (a); $d = 0.55$ in (b).

In the region $(0, \alpha_l]$, the two products are weak substitutes. Sellers have little concern about advertising leakage, and even the highest discoverability level only mildly intensifies price competition. The platform therefore sets $x^* = 1$ to maximize consumers' purchase likelihood, and sellers advertise in equilibrium.

When α increases into the region $(\alpha_l, \alpha_m]$, the platform's optimal discoverability decreases to $x^* < 1$ to maintain external advertising. A larger α aggravates both the ad-discouraging effect (more severe advertising leakage) and the competition-intensifying effect (more switchers and fiercer price competition) caused by a high x. Together, these two forces prompt the platform to reduce x to preserve sellers' advertising incentives and soften price competition, even though doing so lowers consumers' purchase likelihood.

When α further increases into the region $(\alpha_m, \alpha_h]$, the platform would need to set a very low x to incentivize external advertising. However, such a large reduction in discoverability would cause a substantial drop in purchase likelihood, outweighing the benefits from expanded traffic and softer price competition. The platform therefore forgoes incentivizing external advertising and restores $x^* = 1$ to maximize purchase likelihood. Sellers do not advertise in equilibrium.

Finally, when α further increases into the region $(\alpha_h, 1/2)$, the platform's benefit from maximizing purchase likelihood at $x = 1$ diminishes. As noted, a higher α attenuates the match-boosting effect of increasing x, so restricting discoverability entails only a small loss in purchase likelihood. At the same time, a higher α aggravates both the ad-discouraging and competition-intensifying effects, strengthening the platform's incentives to reduce x. Therefore, the platform's incentives to encourage external advertising and alleviate price competition dominate again, leading to $x^* < 1$.

Role of advertising cost-effectiveness. Proposition 2 shows that as external advertising becomes more cost-effective (i.e., as effectiveness d increases or cost k decreases), the platform should initially reduce its discoverability level and then increase it.

Proposition 2 (Advertising cost-effectiveness affects equilibrium product discoverability)**.** *Consider $\lambda = 1$. In equilibrium, the platform limits product discoverability to $x^* < 1$ when seller external advertising has moderate cost-effectiveness ($d_h > d > d_l$ or equivalently $k_l < k < k_h$); otherwise, the platform sets the highest discoverability $x^* = 1$.*

Figure 5(b, first row) illustrates the non-monotonic impact of advertising effectiveness d on the platform's optimal discoverability x^*. The effect of advertising cost k is qualitatively opposite to the effect of d; see Figure W1 in the Web Appendix. Figure 6 provides another illustration. The platform limits product discovery ($x^* < 1$) when external advertising has moderate cost-effectiveness: doing so induces more consumer traffic through seller external advertising. Within this range, as cost-effectiveness increases, sellers have stronger incentives to advertise, so the platform can raise x^* and still motivate sellers to advertise. When cost-effectiveness is very high, sellers advertise regardless of discoverability, so the platform sets $x^* = 1$ to maximize purchase likelihood. When cost-effectiveness is very low, incentivizing advertising would require such low discoverability that the loss in purchase likelihood outweighs the traffic gain, so the platform again sets $x^* = 1$.

Welfare analysis. We now examine welfare implications of the platform's strategic product discoverability choice for different market players. Conventional wisdom suggests two predictions: stronger product substitutes, by intensifying price competition, should hurt sellers while benefiting consumers; and higher advertising cost-effectiveness, by reaching more consumers with less spending, should benefit all market players. The central finding of this section is that *both intuitions fail* once we account for the platform's strategic discoverability choice and the sellers' strategic advertising response. These reactions can reverse conventional wisdom: stronger product substitutes can make sellers better off and consumers worse off, despite fiercer price competition, whereas higher advertising cost-effectiveness can hurt both sellers and consumers. The following proposition formalizes these reversals. Because the two sellers may settle in asymmetric equilibria where only one seller advertises, we use their total profit to evaluate equilibrium payoffs.

Proposition 3 (Welfare outcomes under optimal product discoverability)**.** *Consider $\lambda = 1$. With the platform's optimal product discoverability x^*, in equilibrium:*

(i) When products are more substitutable (a larger α): the platform's profit decreases; however, seller profit can increase, whereas consumer surplus can decrease.

(ii) When external advertising is more cost-effective (a larger d or a smaller k): the platform's profit weakly increases; however, both seller profit and consumer surplus can decrease when product substitutability α is sufficiently low.

Proposition 3 (i) characterizes the welfare effects of higher product substitutability. A higher α reduces the platform's profit because consumers who find one product a mismatch are more likely to also find the other a mismatch, reducing their purchase likelihood on the platform. Mathematically, when consumers discover both products, their purchase likelihood is $(2-\alpha)m$, which decreases in α.

Interestingly, although price competition between sellers is more intense when their products are stronger substitutes, sellers' profit can increase and consumer surplus can decrease. In particular, sellers become better off when α increases from below α_m to above it, prompting the platform to raise product discoverability from $x^* < 1$ to $x^* = 1$. As a consequence, sellers withdraw from external advertising, saving advertising expenses and improving profits. By contrast, consumers become worse off when a higher α prompts the platform to reduce x^*, for example, when α increases within the region (α_l, α_m) or across the threshold α_h. The reduced discoverability lowers consumers' likelihood of finding a matching product and thus their expected surplus. See Figure 5(a, rows three and four) for an illustration.

Proposition 3 (ii) characterizes the welfare impacts of more cost-effective external advertising. Counterintuitively, higher cost-effectiveness can reduce the surplus of both sellers and consumers when product substitutability α is low. As advertising effectiveness d crosses d_l, the platform cuts discoverability from $x^* = 1$ to $x^* < 1$ to induce external advertising. When sellers' products are weak substitutes, they have little overlapping demand, so the discoverability cut barely alleviates competition. The benefit from softer competition and expanded traffic cannot offset the incurred advertising cost, making sellers worse off. The reduced discoverability also diminishes consumers' chances of finding matching products, lowering their surplus. The platform, by contrast, gains through capturing the benefit of expanded external traffic by imposing only a small discoverability cut. Figure 5(b) illustrates these results.[11] These results caution against potential welfare losses from advancements in marketing technologies that enhance advertising effectiveness or reduce costs.

[11]We find a similar welfare result for advertising cost k. When k drops below k_h, the platform lowers discoverability from $x^* = 1$ to $x^* < 1$, reducing seller profit and consumer surplus (see Figure W1 in the Web Appendix).

External advertising does not expand platform traffic ($\lambda = 0$***)***

The preceding analysis has illustrated the traffic-expansion mechanism: the platform profits from restricting discovery to encourage seller external advertising that expands total marketplace traffic. In certain product categories, a seller's external advertising may primarily reach consumers who already know the competing seller and will visit the platform regardless. In such settings, the traffic-expansion benefit of external advertising is limited. Can the platform still benefit from encouraging external advertising?

We examine this question through the polar case $\lambda = 0$, where external advertising does not bring any new traffic to the platform, fully eliminating the traffic-expansion mechanism. We find that the platform can still benefit from limiting discovery, but through a distinct *differentiation* mechanism: restricting discovery induces asymmetric advertising across sellers, and the resulting asymmetry in consumer awareness softens price competition and raises the platform's commission revenue. Proposition 4 formalizes this result.

Proposition 4 (The platform benefits from inducing seller differentiation)**.** *Consider* $\lambda = 0$*. In equilibrium, the platform limits product discoverability to* $x^* < 1$ *if external advertising is sufficiently cost-effective (a small* k *or a large* d*) and products are sufficiently substitutable (a large* α*). In such an equilibrium, one seller invests in external advertising whereas the other seller does not.*

In our model, the differentiation mechanism corresponds to one seller advertising while the other does not: the advertising seller commands a larger "loyal" consumer base than the rival, raising the opportunity cost of a price cut and thereby reducing its incentive to compete aggressively for "switchers." The platform pursues this strategy when products are sufficiently substitutable (so softening competition yields a significant benefit) and external advertising is sufficiently cost-effective (so inducing advertising does not require too large a reduction in discoverability). Notably, we find that the platform *never* limits discoverability to induce both sellers to advertise in equilibrium, because doing so neither expands consumer traffic nor creates seller differentiation. As a side remark, in the earlier discussion of Proposition 1, we noted that limiting discoverability itself can alleviate price competition by reducing the number of "switchers." Here, Proposition 4 highlights a distinct channel: inducing asymmetric advertising creates unequal "loyal" consumer bases across sellers, which softens competition through seller differentiation rather than through a reduction in switchers.

The differentiation mechanism yields an interesting implication for the platform. When external advertising becomes more costly, sellers are less willing to advertise, which would seem to hurt the platform, because the platform needs to further limit product discovery to maintain advertising, and the tighter limit reduces consumers' purchase likelihood. However, Corollary 1 shows that the platform's equilibrium profit can in fact rise.

Corollary 1 (Platform profit under differentiation mechanism)**.** *Consider $\lambda = 0$. As advertising cost k increases, the platform's equilibrium profit can increase.*

The reason is that asymmetric advertising benefits the platform by creating differentiation in the number of consumers who eventually know about each seller. This differentiation diminishes as discoverability x increases: when x approaches one, all consumers discover both products on the platform even if only one seller advertises.[12] Therefore, conditional on inducing asymmetric advertising, the platform has an incentive to lower x to further exploit the benefit from seller differentiation (despite the resulting reduction in consumers' purchase likelihood). However, this reduction in x is constrained by the requirement that x not be so low as to induce both sellers to advertise, which would eliminate the benefit

[12] Mathematically, when only seller A advertises but seller B does not, equations (2)–(3) imply that the two sellers' loyal consumer bases differ by $n_A - n_B = d(1-x)m$, which decreases with x.

from differentiation. This constraint relaxes as advertising becomes more costly, so the platform can set a lower x^* and earn a higher profit in certain parameter regions.[13]

General external advertising: $\lambda \in (0, 1)$

So far, we have investigated two polar cases of $\lambda = 0, 1$ to illustrate two distinct incentives for the platform to limit product discoverability: expanding consumer traffic and inducing seller differentiation. For a general $\lambda \in (0, 1)$, a seller's external advertising reaches a mix of consumers who are unaware of any product and consumers who already know the competing product. We provide numerical analysis in the Web Appendix. All the key qualitative insights from both polar cases can hold. We also illustrate how the equilibrium regimes of product discoverability and external advertising evolve as λ varies between the two polar cases.

Managerial implications. Our analysis reveals a strategic trade-off for an e-commerce platform when designing its product discovery features. Although prior literature and industry practice emphasize how product discovery tools benefit the platform by enhancing consumers' purchase likelihood (eMarketer, 2021; Wan, Kumar, and Li, 2024), such tools can inadvertently undermine the platform's profitability by discouraging sellers from investing in external advertising. Platforms should therefore recognize this unintended consequence and account for sellers' strategic responses when designing these features.

Our findings further suggest several considerations for tailoring product discovery across product categories. First, platforms should consider how similar competing products are and how mature the product market is. In emerging markets where consumers are not yet familiar with most sellers, such as robot vacuums and other innovative smart home devices, platforms can benefit from limiting product discovery features when competing

[13] For how the platform's profit changes with k in other regions, see Figure W3 and its accompanying note in the Web Appendix.

products are either moderately differentiated or highly similar. In mature markets where most consumers already know competing sellers, such as smartphones or athletic shoes, platforms can benefit from limiting discovery features when competing products are highly similar. Second, platforms can consider the overall appeal of product categories. While platforms can benefit from extensive product discovery features in categories with mass-market or narrow-niche appeal, limiting discovery features can be profitable in categories of moderate appeal.[14] Finally, when new marketing technologies such as generative AI ad creative improve external advertising's returns, platforms should be mindful of how their product discovery features interact with sellers' evolving advertising incentives.

EXTENSIONS

Endogenous Commission Rate

In the main model, we fix the platform's commission rate r to focus on its decision on product discoverability. We now let the platform endogenously decide both r and x.

We find that, beyond limiting product discoverability, the platform can also lower its commission rate to encourage seller external advertising, trading off the commission revenue collected per consumer against the additional external traffic that advertising brings.[15] This implies that the platform has two strategic levers to facilitate external advertising, lowering discoverability x and reducing commission rate r. The following proposition characterizes the joint optimum. We present the result for $\lambda = 1$ here, though the qualitative insight holds for general λ.[16]

[14] As noted earlier, sellers are more incentivized to advertise when consumers are more likely to find their products a match (i.e., when the products' match likelihood m is high).

[15] In the Web Appendix, we show that when sellers do not implement external advertising, the platform would set the commission rate to $r = 1$ to extract surplus from sellers. By contrast, when sellers strategically decide on external advertising, the platform may instead set $r < 1$ to incentivize their investment. The result of $r = 1$ is an artifact of our normalization that sellers have zero marginal production cost and zero outside option. Our qualitative insights regarding the trade-off do not rely on these two assumptions.

[16] The analysis is given in the Web Appendix, where we also show the result for $\lambda = 0$ and a numerical analysis in Figure W5 with $\lambda = 0.5$. Results for general λ are qualitatively similar.

Proposition 5 (Platform endogenously decides both product discoverability x and commission rate r)**.** *Consider $\lambda = 1$. The platform limits product discoverability to $x^* < 1$ and commission rate to $r^* < 1$ when external advertising has moderate cost-effectiveness ($\widetilde{d}_l < d < \widetilde{d}_h$ or equivalently $\widetilde{k}_h > k > \widetilde{k}_l$), in which case at least one seller invests in external advertising in equilibrium.*

Proposition 5 shows that the platform can combine both levers, lowering product discoverability and reducing commission rate, to further incentivize external advertising and improve its commission revenue. Importantly, even when the platform can adjust commission rate, limiting product discoverability remains a profitable strategic instrument, demonstrating the robustness of our core insight. Moreover, consistent with our main model finding, the platform benefits from limiting product discoverability when seller external advertising has moderate cost-effectiveness (see Proposition 2).

In practice, the reduced-commission lever also resonates with the external traffic programs we discussed in the Introduction. Amazon's Brand Referral Bonus, Walmart's Sales Rewards & Attribution, and Etsy's Share & Save programs return to sellers a percentage of qualifying sales attributed to external traffic, effectively lowering the platform's net commission on those sales. Our analysis provides an explanation for these programs as a real-world counterpart to the commission-reduction lever in our model.

Other Robustness Checks

We have also extended our analyses in several settings to demonstrate the robustness of our key insights. A brief discussion is provided below, with further details relegated to the Web Appendix.

Platform visitors unaware of any product. The main model assumes that every consumer visiting the platform already knows at least one seller's product. In practice, some consumers may visit the platform to browse a product category without a specific seller in mind. Product discoverability on the platform makes these consumers more likely to find products that match their needs. Specifically, we assume that a mass $\eta \geq 0$ of consumers visit the platform without knowing any seller's product. Product discoverability allows a fraction x of these unaware consumers to discover both products, and the rest do not discover any product. Intuitively, higher discoverability helps convert unaware consumers into buyers, so as η grows, the platform has a stronger incentive to raise x. Our key insights are robust provided η is not too large; in particular, the platform still finds it optimal to limit discoverability to encourage external advertising.

Continuous advertising levels. The main model treats each seller's external advertising decision as binary: pay investment cost k or forgo advertising entirely. This formulation captures settings where advertising takes the form of a single campaign decision. Our core insights extend to an alternative specification in which sellers choose continuous advertising levels, accommodating settings where budgets can be adjusted more flexibly. Specifically, each seller chooses an advertising intensity $y_j \in [0, 1]$ at a convex cost $k(y_j + y_j^2)$, with higher intensity raising the probability of achieving high awareness.

Advertising persuasion effect. The main model treats seller external advertising as purely informative: it expands consumer awareness without affecting product valuations. In practice, advertising may also serve a persuasive function, raising consumers' valuations for advertised products. A stronger persuasion effect increases sellers' incentives to advertise. We model this effect by allowing a seller's external advertising to increase its product match probability m by $\gamma \geq 0$ for consumers reached by the advertising. Our key insight that the platform encourages external advertising through reduced discoverability remains valid as long as the persuasion effect is not so strong that sellers would always advertise regardless of discoverability.

Exposure to both sellers' ads. The main model assumes that when both sellers advertise, their ad-reached consumers are disjoint. In practice, some consumers may encounter both sellers' ads and learn both products before visiting the platform. To capture this, we introduce an overlap parameter $\delta \in [0, 1]$. When both sellers advertise, each seller's ad still reaches a mass λd of consumers who initially know neither product. Among them, $\delta\lambda d$ encounter both sellers' ads and learn both products, whereas $(1-\delta)\lambda d$ encounter only that seller's ad. Our key qualitative insights remain robust under this extension.

CONCLUDING REMARKS

External traffic has become an increasingly important source of demand for e-commerce marketplaces. As consumers' shopping journeys more frequently begin off-platform, seller external advertising has emerged as a scalable way to bring incremental shoppers into marketplaces. Yet external traffic does not arrive in a vacuum: once consumers land on a marketplace, the platform's product discovery environment shapes what they see, what they buy, and, crucially, how much value an advertising seller can appropriate from the traffic it attracts. We develop a game-theoretic model to investigate how a commission-based marketplace should design product discovery when competing sellers can invest in external advertising. Our central insight is that product discovery design is not merely a conversion tool but also an instrument that governs advertising leakage and therefore sellers' incentives to strategically invest in external advertising. We show that a platform may benefit from limiting product discovery: although higher discoverability boosts shoppers' purchase likelihood, it can weaken sellers' incentives to advertise externally, thereby reducing the platform's commission revenue.

We identify two distinct mechanisms through which limiting discovery benefits the platform: a *traffic-expansion* mechanism, where limiting discovery mitigates advertising leakage and encourages sellers to advertise externally, expanding platform traffic; and a *differentiation* mechanism, where limiting discovery induces asymmetric advertising and softens price competition. We further characterize how optimal discoverability responds to market conditions, including non-monotonic patterns in product substitutability and in the cost-effectiveness of external advertising, reflecting the platform's balance between traffic expansion and purchase likelihood. The platform's strategic decision on product discoverability also yields counterintuitive welfare implications: higher product substitutability between competing sellers, despite intensifying their price competition, may increase seller profit and reduce consumer surplus, whereas higher advertising cost-effectiveness for sellers can reduce their profit and consumer surplus. In addition, we show that the platform can complement restricted discoverability with a lower commission rate to further encourage seller external advertising and improve its commission revenue.

Our paper offers directions for future research. First, we focus on a market where sellers are horizontally differentiated. Future research can investigate how product discoverability affects vertically differentiated sellers differently depending on their product quality, and how the platform should set discoverability in such a market. Second, we study a platform with an agency selling model, where sellers set final prices and pay a commission to the platform. Future research can examine the discoverability strategy of a platform with a reselling model, where the platform acquires products from sellers and resells them to consumers at retail prices determined by the platform (Abhishek, Jerath, and Zhang, 2016). Third, future research can explore how the platform's discoverability decision might interact with sellers' competition on external advertising platforms (e.g., Google). Finally, future research can investigate the interaction between sellers' external advertising and their on-site sponsored advertising.

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Web Appendix
Product Discoverability in E-Commerce Marketplaces with Seller External Advertising

Disclosure: These materials have been supplied by the authors to aid in the understanding of their paper. The AMA is sharing these materials at the request of the authors.

CONTENTS

ADDITIONAL TABLES AND FIGURES

Table W1: Consumer demand composition

Seller advertising	A-loyals (n_A)
$I_A = I_B = N$	$(1+x-2x\alpha)m$
$I_A = Y,\ I_B = N$	$\Big(1+d+x-(2+d)x\alpha\Big)m - dm(\alpha(1-2x)+x)(1-\lambda)$
$I_A = N,\ I_B = Y$	$\Big(1+(1+d)x-(2+d)x\alpha\Big)m - dm(\alpha(1-2x)+x)(1-\lambda)$
$I_A = I_B = Y$	$(1+d)(1+x-2x\alpha)m - 2dm(\alpha(1-2x)+x)(1-\lambda)$

Seller advertising	B-loyals (n_B)
$I_A = I_B = N$	$(1+x-2x\alpha)m$
$I_A = Y,\ I_B = N$	$\Big(1+(1+d)x-(2+d)x\alpha\Big)m - dm(\alpha(1-2x)+x)(1-\lambda)$
$I_A = N,\ I_B = Y$	$\Big(1+d+x-(2+d)x\alpha\Big)m - dm(\alpha(1-2x)+x)(1-\lambda)$
$I_A = I_B = Y$	$(1+d)(1+x-2x\alpha)m - 2dm(\alpha(1-2x)+x)(1-\lambda)$

Seller advertising	Switchers (n_{AB})	Total demand ($n_A + n_B + n_{AB}$)
$I_A = I_B = N$	$2x\alpha m$	$2m(1+x-\alpha x)$
$I_A = Y,\ I_B = N$	$(2+d)x\alpha m + \alpha dm(1-2x)(1-\lambda)$	$(2+d)m(1+x-\alpha x) - dm(2(1-\alpha)x+\alpha)(1-\lambda)$
$I_A = N,\ I_B = Y$	$(2+d)x\alpha m + \alpha dm(1-2x)(1-\lambda)$	$(2+d)m(1+x-\alpha x) - dm(2(1-\alpha)x+\alpha)(1-\lambda)$
$I_A = I_B = Y$	$(2+2d)x\alpha m + 2\alpha dm(1-2x)(1-\lambda)$	$2(1+d)m(1+x-\alpha x) - 2dm(2(1-\alpha)x+\alpha)(1-\lambda)$

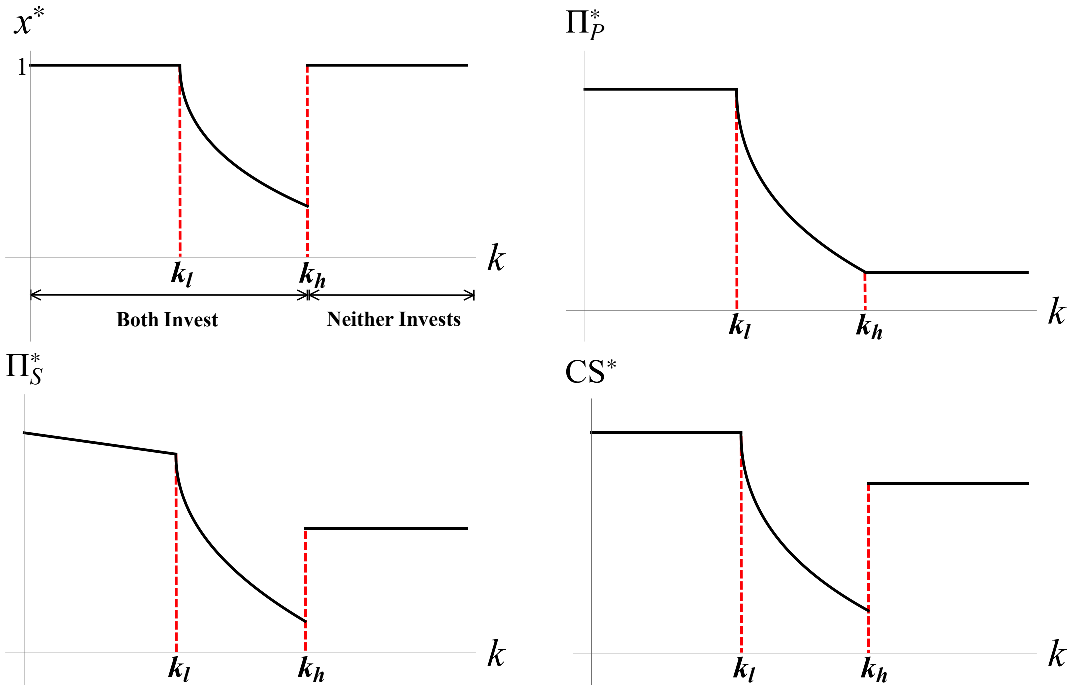


Figure W1: Effects of k on equilibrium outcomes under $\lambda = 1$

Note. $v = 2$, $r = 0.5$, $m = 0.4$, $d = 0.3$, and $\alpha = 0.3$.

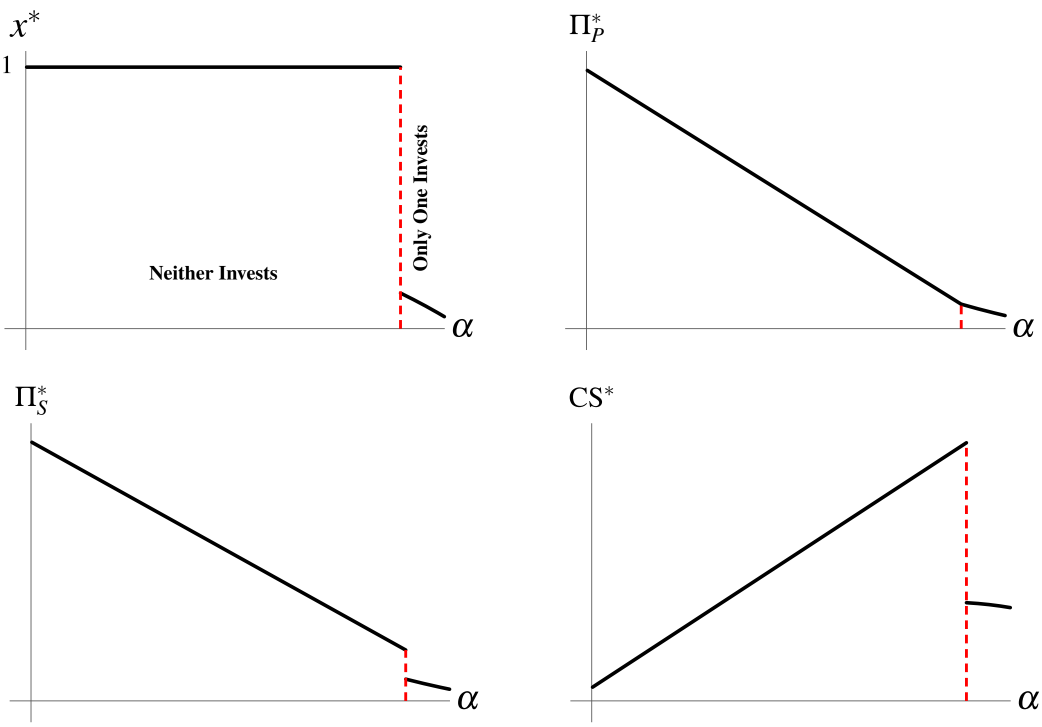


Figure W2: Effects of α on equilibrium outcomes under $\lambda = 0$

Note. $v = 2$, $r = 0.5$, $m = 0.4$, $d = 0.6$, and $k = 0.1$.

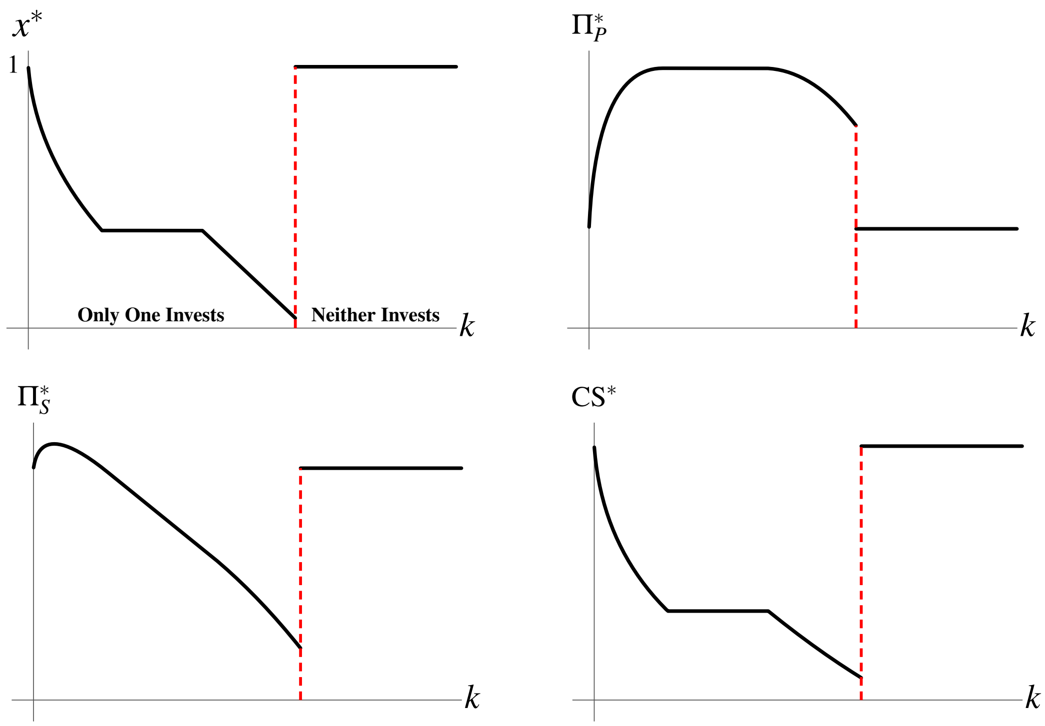


Figure W3: Effects of k on equilibrium outcomes under $\lambda = 0$

Note. $v = 2$, $r = 0.5$, $m = 0.4$, $d = 0.6$, and $\alpha = 0.48$. Consistent with Corollary 1, the platform's equilibrium profit can initially rise with k. As k rises further, x^* becomes so low that the loss in purchase likelihood dominates the differentiation benefit, and the platform's profit declines. When k becomes very high, the platform abandons inducing advertising and sets full discoverability, and its profit remains constant. The figure also illustrates counterintuitive impacts of k on consumer surplus and seller profit, similar to those in Proposition 3.

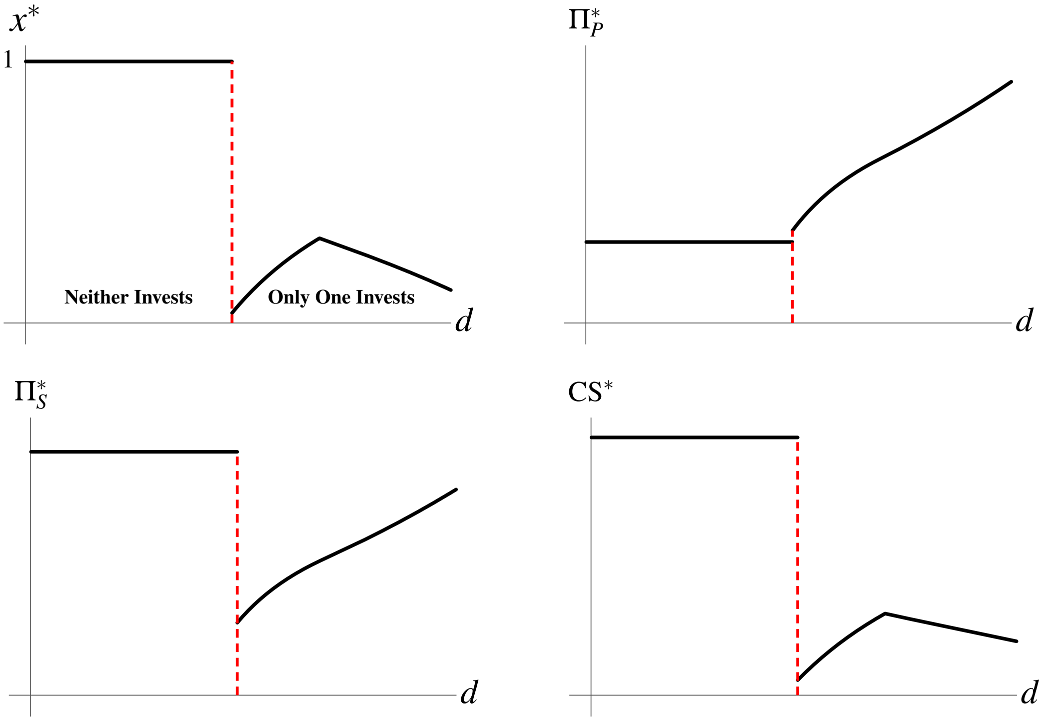
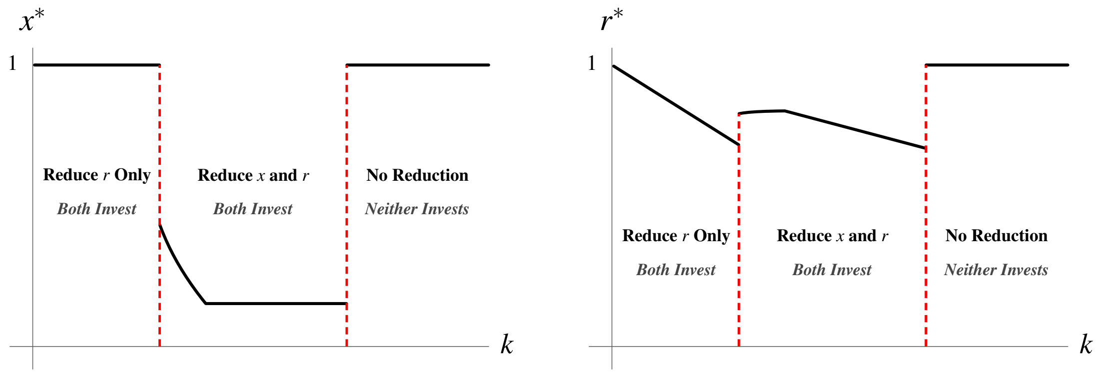


Figure W4: Effects of d on equilibrium outcomes under $\lambda = 0$

Note. $v = 2$, $r = 0.5$, $m = 0.4$, $\alpha = 0.48$, and $k = 0.1$.



(a) Optimal product discoverability x^* (b) Optimal commission rate r^*

Figure W5: Equilibrium product discoverability and commission rate

Note. $v = 2$, $m = 0.4$, $\alpha = 0.4$, $d = 0.8$, and $\lambda = 0.5$.

PROOFS AND ANALYSIS FOR MAIN MODEL RESULTS

Proof of Sellers' Pricing Equilibrium at Stage 3

We derive the pricing equilibrium at Stage 3 where the sellers' external advertising decisions and the platform's product discoverability choice are given. Given the fixed commission rate r applied equally to both sellers, the pricing subgame is similar to Narasimhan (1988) in the sense that the size of loyal consumers for seller j is n_j, the size of switchers is n_{AB}, and the consumers' reservation price is v. Below, we solve the equilibrium putting aside the percentage commission charged by the platform and the advertising cost k, but they will be included in the eventual equilibrium profits.

Suppose seller j has a (weakly) larger size of loyal consumers than its competitor seller $-j$, i.e., $n_j \geq n_{-j}$. Following Narasimhan (1988), the subgame has no pure-strategy pricing equilibrium but a unique mixed-strategy pricing equilibrium, in which seller j randomizes p_j on a continuous distribution on an interval $[\overline{p}, v]$ with cumulative distribution $F_j(p)$ with a mass point at $p_j = v$, and seller $-j$ randomizes p_{-j} on a continuous distribution on an interval $[\overline{p}, v)$ with cumulative distribution $F_{-j}(p)$, where

$$\overline{p} = v\frac{n_j}{n_j + n_{AB}}. \tag{W1}$$

Following Narasimhan (1988), in such a mixed-strategy equilibrium, seller j receives the same expected profit for any $p_j \in [\overline{p}, v]$, given that seller $-j$'s price p_{-j} follows the equilibrium distribution $F_{-j}(p)$. Specifically, its equilibrium profit equals vn_j, which is its profit when seller j charges $p_j = v$ and sells only to its own loyal consumers, whose population is n_j. The equilibrium price distribution $F_{-j}(p)$ can be derived as

$$vn_j = p_j \left[F_{-j}(p_j)n_j + (1 - F_{-j}(p_j))(n_j + n_{AB})\right] \Rightarrow F_{-j}(p) = 1 - \frac{n_j(v-p)}{n_{AB} \cdot p}. \tag{W2}$$

Similarly, seller $-j$'s profit when charging $p_{-j} = \overline{p}$ equals $\overline{p}(n_{-j} + n_{AB})$, so $F_j(p)$ can be derived as

$$\overline{p}(n_{-j}+n_{AB}) = p_{-j}[F_j(p_{-j})n_{-j}+(1-F_j(p_{-j}))(n_{-j}+n_{AB})] \Rightarrow F_j(p) = 1+\frac{n_{-j}}{n_{AB}}-\frac{n_j(n_{-j}+n_{AB})v}{n_{AB}(n_j+n_{AB})p}. \tag{W3}$$

Note that $F_j(p)$ has a mass point at the reservation price v with probability $\frac{n_j-n_{-j}}{n_j+n_{AB}}$. The values of n_A, n_B, n_{AB} depend on sellers' advertising decisions given by Table W1. Table W2 summarizes the equilibrium profits under different advertising scenarios. □

Table W2: Equilibrium seller profits from product sales

Seller advertising	Seller profits
$I_A = I_B = N$	$\Pi_j = (v - vr)n_j$
$I_A = I_B = Y$	$\Pi_j = (v - vr)n_j - k$
$I_j = Y, I_{-j} = N$	$\Pi_j = (v - vr)n_j - k$
$(j = A, B)$	$\Pi_{-j} = (v - vr)\frac{n_j}{n_j+n_{AB}}(n_{-j} + n_{AB})$

Proof of Lemma 1: Sellers' Advertising Equilibrium at Stage 2

Lemma 1 summarizes the equilibrium of the seller external advertising subgame at Stage 2. In part (1) of this proof, we derive how the sellers' equilibrium advertising decisions are affected by the advertising cost k. In part (2), we derive how these decisions are affected by the advertising effectiveness d. Note that the conditions in the two parts can be equivalently translated into each other.

Part (1): with respect to k. Table W3 presents the payoff matrix for the seller external advertising subgame, where the detailed expressions for these payoffs are given in Table W2. Note that $\Pi_B^{YN} = \Pi_A^{NY}$ and $\Pi_A^{YN} = \Pi_B^{NY}$.

Table W3: Payoff matrix for the seller external advertising subgame

	Seller B ($I_B = N$)	Seller B ($I_B = Y$)
Seller A ($I_A = N$)	Π_A^{NN}, Π_B^{NN}	Π_A^{NY}, Π_B^{NY}
Seller A ($I_A = Y$)	Π_A^{YN}, Π_B^{YN}	Π_A^{YY}, Π_B^{YY}

A pure-strategy equilibrium can take one of the three forms: YY, where both sellers invest in external advertising; YN or NY, where only one seller invests in external advertising; and NN, where no seller invests in external advertising.

YY is an equilibrium if and only if $\Pi_A^{YY} \geq \Pi_A^{NY}$ (or equivalently $\Pi_B^{YY} \geq \Pi_B^{YN}$), which yields

$$\begin{aligned} k \leq \widehat{k}(x) \equiv \frac{(v - vr)md}{1 + x + d + (\lambda - 1)xd} \Big(& \alpha(2d+1)(\lambda - 1) + d + 1 + \\ & x \left(\alpha \left(d \left(\lambda^2 - 7\lambda + 4\right) - \lambda - 2\right) + 2d(\lambda - 1) + \lambda\right) + \\ & x^2 \left(\alpha \left(d \left(-2\lambda^2 + 5\lambda - 2\right) - 2\lambda + 3\right) + (\lambda - 1)(d(\lambda - 1) + 1)\right) \Big). \end{aligned} \quad \text{(W4)}$$

NN is an equilibrium if and only if $\Pi_A^{NN} \geq \Pi_A^{YN}$ (or equivalently $\Pi_B^{NN} \geq \Pi_B^{NY}$), which yields

$$k \geq \overline{k}(x) \equiv (v - vr)md\Big(\alpha(\lambda - 2\lambda x + x - 1) + (\lambda - 1)x + 1\Big). \quad \text{(W5)}$$

Finally, YN or NY is an equilibrium if and only if $\Pi_A^{NN} \leq \Pi_A^{YN}$ and $\Pi_B^{YN} \geq \Pi_B^{YY}$ (or equivalently $\Pi_B^{NN} \leq \Pi_B^{NY}$ and $\Pi_A^{NY} \geq \Pi_A^{YY}$), which yields

$$\overline{k}(x) \geq k \geq \widehat{k}(x). \quad \text{(W6)}$$

Note that $\overline{k}(x) - \widehat{k}(x) = \frac{\alpha m d(v-vr)(1-x)(d+2x+(2\lambda-1)xd-d\lambda)}{1+x+d+(\lambda-1)xd} \geq 0$. Moreover, one can verify that both $\widehat{k}(x)$ and $\overline{k}(x)$ decrease in x (strictly so for $x \in [0,1)$), and

$$\widehat{k}(1) = \overline{k}(1) = (1-\alpha)(v-vr)md\lambda \equiv k_1, \tag{W7}$$

$$\widehat{k}(0) = \frac{(v-vr)md(d+1+\alpha(2d+1)(\lambda-1))}{d+1} \equiv k_{12}, \tag{W8}$$

$$\overline{k}(0) = (v-vr)md(1-\alpha+\alpha\lambda) \equiv k_2, \tag{W9}$$

where $k_1 < k_{12} \leq k_2$. Next, we discuss the pure-strategy equilibrium outcome based on the relative magnitudes between k, k_1, k_{12}, and k_2.

Case (i): $k < k_1$. In this region, $k < \widehat{k}(x)$ holds for all $x \in [0,1]$, so the unique pure-strategy equilibrium is YY, i.e., both sellers invest.

Case (ii): $k_1 < k < k_{12}$. In this region, because both $\widehat{k}(x)$ and $\overline{k}(x)$ strictly decrease with x, there exist unique $0 < x_1 < x_2 < 1$ such that $k = \widehat{k}(x_1) = \overline{k}(x_2)$. Based on equations (W4)-(W6), the pure-strategy equilibrium is YY when $0 \leq x < x_1$, is YN or NY when $x_1 < x < x_2$, and is NN when $x_2 < x \leq 1$.

Case (iii): $k_{12} < k < k_2$. In this region, $k > \widehat{k}(x)$ holds for all $x \in [0,1]$, so YY is not an equilibrium. Since $\overline{k}(x)$ strictly decreases with x, there exists a unique $0 < x_2 < 1$ such that $k = \overline{k}(x_2)$. Based on equations (W4)-(W6), the pure-strategy equilibrium is YN or NY when $0 \leq x < x_2$, and is NN when $x_2 < x \leq 1$.

Case (iv): $k > k_2$. In this region, $k > \overline{k}(x)$ holds for all $x \in [0,1]$, so the unique pure-strategy equilibrium is NN, i.e., no seller invests.

Lemma 1 provides a more concise exposition by merging cases (ii) and (iii), and states that when $k_1 < k < k_2$, the equilibrium is YY when $x < x_1$, is YN or NY when $x_1 < x < x_2$, and is NN when $x > x_2$. For this purpose, we define $x_1 = 0$ in case (iii), whereas x_1 solves $k = \widehat{k}(x_1)$ in case (ii). Note that x_2 solves $k = \overline{k}(x_2)$ in both cases.

Part (2): with respect to d. We further show that the above conditions can be equivalently converted into conditions with respect to d. To see this, note that both $\widehat{k}(x)$ and $\overline{k}(x)$ strictly increase with d. Therefore, $k \lesseqgtr \widehat{k}(x)$ can be equivalently written as $d \gtreqless \widehat{d}(x)$, and $k \lesseqgtr \overline{k}(x)$ can be equivalently written as $d \gtreqless \overline{d}(x)$.

Therefore, YY is an equilibrium if and only if $d \geq \widehat{d}(x)$, NN is an equilibrium if and only if $d \leq \overline{d}(x)$, and YN or NY is an equilibrium if and only if $\overline{d}(x) \leq d \leq \widehat{d}(x)$.

Moreover, $\overline{d}(x) \leq \widehat{d}(x)$; both $\widehat{d}(x)$ and $\overline{d}(x)$ strictly increase with x. Then, following similar steps in Part (1), we can derive the conditions about d. Note that k_1 and d_2, and likewise k_2 and d_1, are defined by the same equation; $k \leq k_1$ is equivalent to $d \geq d_2$, and $k \geq k_2$ is equivalent to $d \leq d_1$. $\square$

Proof of Lemma 2: Optimal Discoverability Without Seller Advertising

When no seller advertises, the platform's total commission $\Pi_P(x) = 2vmr(1 + (1 - 2\alpha)x)$ increases with x on $[0, 1]$ given $\alpha < 1/2$, hence $x^* = 1$. $\square$

Proof of Propositions 1 and 2: Optimal Discoverability When $\lambda = 1$

When $\lambda = 1$, $k_{12} = k_2$ in the proof of Lemma 1. Below we derive the equilibrium at Stage 1, where the platform sets the discoverability level $x \in [0, 1]$ to maximize its total profit:

$$\Pi_P(x) = \begin{cases} 2vmr(1 + (1 - 2\alpha)x) & \text{if } k \geq k_2, \text{ or if } k_1 < k < k_2 \text{ and } x > x_2, \\ \frac{(2+d)vmr(1+x)(1+(1-2\alpha)x+d(1-\alpha x))}{1+d+x} & \text{if } k_1 < k < k_2 \text{ and } x_1 < x \leq x_2, \\ (2 + 2d)vmr(1 + (1 - 2\alpha)x) & \text{if } k \leq k_1, \text{ or if } k_1 < k < k_2 \text{ and } x \leq x_1, \end{cases} \tag{W10}$$

where

$$
\begin{aligned}
&k_1 \equiv (1-\alpha)(v-vr)md,\\
&k_2 \equiv (v-vr)md,\\
&x_1 \equiv \frac{1}{2(1+d)(v-vr)md\alpha}\Big((3\alpha+2d\alpha-1)(v-vr)md+k-\\
&\quad \sqrt{((3\alpha+2d\alpha-1)(v-vr)md+k)^2-4((v-vr)md-k)(1+d)^2(v-vr)md\alpha}\Big),\\
&x_2 \equiv \frac{(v-vr)md-k}{(v-vr)md\alpha}.
\end{aligned}
\tag{W11}
$$

We first derive the results with respect to k in Part (1). The analysis for d is very similar to that for k, so we only briefly highlight the key thresholds at the end of Part (1). Then, we prove the results regarding α in Part (2).

Part (1): with respect to k. The result is summarized in Lemma W1 below.

Lemma W1 (Optimal product discoverability when $\lambda = 1$)**.** *There exist thresholds $k_l \le k_h$ such that (a) if $k \le k_l$, the platform sets $x^* = 1$, and both sellers invest; (b) if $k \ge k_h$, the platform sets $x^* = 1$, and neither seller invests; (c) if $k_l < k < k_h$, the platform limits $x^* < 1$, and at least one seller invests.*

Proof: To prove Lemma W1, based on equation (W10), we divide the analysis into three parameter regions: (i) $k \le k_1$, (ii) $k \ge k_2$, and (iii) $k_1 < k < k_2$.

Region (i): $k \le k_1$. In this region, both sellers will invest, so $\Pi_P(x) = (2+2d)vmr(1+(1-2\alpha)x)$, $\forall x \in [0,1]$. Given $\alpha < 1/2$, the platform will choose $x^* = 1$, and both sellers invest in equilibrium.

Region (ii): $k \ge k_2$. In this region, neither seller will invest, hence $\Pi_P(x) = 2vmr(1+(1-2\alpha)x)$, $\forall x \in [0,1]$. Given $\alpha < 1/2$, the platform will choose $x^* = 1$, and neither seller invests in equilibrium.

Region (iii): $k_1 < k < k_2$. In this region, the platform's profit function is reduced to

$$\Pi_P(x) = \begin{cases} 2vmr(1+(1-2\alpha)x) & \text{if } x > x_2, \\ \frac{(2+d)vmr(1+x)(1+(1-2\alpha)x+d(1-\alpha x))}{1+d+x} & \text{if } x_1 < x \le x_2, \\ (2+2d)vmr(1+(1-2\alpha)x) & \text{if } x \le x_1. \end{cases} \tag{W12}$$

In this case, one can see that the platform's profit increases with x within the interval $x_2 < x \le 1$. Therefore, if the platform were not to induce external advertising, it would choose $x = 1$. We will show that there exists $k_h \in (k_1, k_2)$ such that the platform chooses $x^* < 1$ to induce *at least one seller* to advertise when $k_1 < k < k_h$, whereas the platform chooses $x^* = 1$ to *not* induce external advertising when $k_h < k < k_2$, where

$$k_h = \begin{cases} (v-vr)md & \text{if } \frac{1-d}{2} \le \alpha < \frac{1}{2}, \\ \max\{k_{h1}, k_{h2}\} & \text{if } 0 < \alpha < \frac{1-d}{2}. \end{cases} \tag{W13}$$

We first show that x_1 and x_2 decrease with k. Clearly, x_2 decreases with k. Differentiation yields $\frac{\partial x_1}{\partial k} = \frac{2\sqrt{(k-(1-\alpha)(v-vr)md)((\alpha(3+2d)^2-1)(v-vr)md+k)}-2\left(\left(\alpha(6d+2d^2+5)-1\right)(v-vr)md+k\right)}{4\alpha(1+d)(v-vr)md\sqrt{(k-(1-\alpha)(v-vr)md)((\alpha(3+2d)^2-1)(v-vr)md+k)}}$ and $\frac{\partial x_1}{\partial k} < 0 \Leftrightarrow -16\alpha^2\left(d^2+3d+2\right)^2(v-vr)^2m^2d^2 < 0$.

We then divide the discussion into two cases: $\frac{1-d}{2} \le \alpha < \frac{1}{2}$ and $0 < \alpha < \frac{1-d}{2}$.

Case 1: $\frac{1-d}{2} \le \alpha < \frac{1}{2}$. In this case, one can see that $\Pi_P(x)$ increases with x within the intervals $0 \le x \le x_1$ and $x_2 < x \le 1$, respectively. We will show that $\Pi_P(x_1) > \Pi_P(1)$ for $k_1 < k < k_2$, implying that the platform must choose $x^* < 1$ rather than $x^* = 1$ when $k_1 < k < k_2$ and $\frac{1-d}{2} \le \alpha < \frac{1}{2}$. As such, we let $k_h = (v-vr)md$ in equation (W13).

Note that $\Pi_P(x_1)$ decreases with k for $k_1 < k < k_2$, as $\Pi_P(x)$ increases with x and x_1 decreases with k, whereas $\Pi_P(1)$ does not vary with k. One can show that $[\Pi_P(x_1) - \Pi_P(1)]|_{k=k_2} = 2vmr(2\alpha+d-1)$, which is non-negative when $\alpha \ge \frac{1-d}{2}$. This is sufficient to prove Case 1: the platform sets $x^* < 1$. In fact, one can further verify that $\Pi_P(x_1) \geq \Pi_P(x)$ for all $x \in (x_1, x_2]$ as well, so the platform optimally chooses $x^* = x_1$, and both sellers invest in equilibrium.

Case 2: $0 < \alpha < \frac{1-d}{2}$. In this case, one can verify that $\Pi_P(x)$ increases with x within all the three intervals $x_2 < x \leq 1$, $x_1 < x \leq x_2$, and $0 \leq x \leq x_1$, respectively. Therefore, the platform's profit maximization problem can be reduced to choosing x among $\{x_1, x_2, 1\}$: if the platform chooses $x = x_1$, it induces both sellers to invest (see the bottom piece in equation (W12)); if the platform chooses $x = x_2$, it induces only one seller to invest (see the middle piece in equation (W12)); if the platform chooses $x = 1$, it induces no sellers to invest (see the top piece in equation (W12)). We proceed to investigate $\max\{\Pi_P(x_1), \Pi_P(x_2)\} > \Pi_P(1)$, which implies that the platform must choose $x^* < 1$ rather than $x^* = 1$.

We first compare $\Pi_P(x_1)$ with $\Pi_P(1)$. Note that $\Pi_P(x_1)$ decreases with k on (k_1, k_2). Therefore, $\Pi_P(x_1)|_{k=k_1} = 4(1-\alpha)(1+d)mvr > \Pi_P(x_1) > \Pi_P(x_1)|_{k=k_2} = 2(1+d)mvr$. By contrast, $\Pi_P(1) = 4(1-\alpha)mvr$ does not change with k. Clearly, $\Pi_P(x_1)|_{k=k_1} > \Pi_P(1)$. Given $\alpha < \frac{1-d}{2}$, $\Pi_P(x_1)|_{k=k_2} < \Pi_P(1)$. So, there exists $k_{h1} \in (k_1, k_2)$ such that $\Pi_P(x_1) > \Pi_P(1)$ when $k_1 < k < k_{h1}$ and $\Pi_P(x_1) < \Pi_P(1)$ when $k_{h1} < k < k_2$, where

$$k_{h1} = \frac{\left(8\alpha^2(d+1) - 2\alpha(3d+4) + d^2 + d + 2\right)(1-\alpha)(v - vr)md}{(1-2\alpha)\left(d^2 + d + 2 - 2\alpha\left(d^2 + 2d + 2\right)\right)} \tag{W14}$$

is the unique solution to $\Pi_P(x_1) = \Pi_P(1)$.

We next compare $\Pi_P(x_2)$ with $\Pi_P(1)$. Likewise, $\Pi_P(x_2)$ decreases with k on (k_1, k_2). Therefore, $\Pi_P(x_2)|_{k=k_1} = 2(1-\alpha)(2+d)mvr > \Pi_P(x_2) > \Pi_P(x_2)|_{k=k_2} = (2+d)mvr$. Clearly, $\Pi_P(x_2)|_{k=k_1} > \Pi_P(1)$. Given $\alpha < \frac{1-d}{2}$, $\Pi_P(x_2)|_{k=k_2} < \Pi_P(1)$. So, there exists $k_{h2} \in (k_1, k_2)$ such that $\Pi_P(x_2) > \Pi_P(1)$ when $k_1 < k < k_{h2}$ and $\Pi_P(x_2) < \Pi_P(1)$ when

$k_{h2} < k < k_2$, where

$$k_{h2} = \frac{(v - vr)md}{2(d+2)(\alpha(d+2)-1)}\Big((\alpha d + d + 2)(\alpha(d+4)-2)+$$
$$\alpha\sqrt{\alpha^2(d(d(d(d+24)+96)+128)+64)+2\alpha(d(d((d-2)d-28)-48)-32)+(d(d+2)+4)^2}\Big) \tag{W15}$$

is the unique solution to $\Pi_P(x_2) = \Pi_P(1)$.

Together, we show that $\max\{\Pi_P(x_1), \Pi_P(x_2)\} > \Pi_P(1)$ when $k_1 < k < \max\{k_{h1}, k_{h2}\}$ and $\max\{\Pi_P(x_1), \Pi_P(x_2)\} < \Pi_P(1)$ when $\max\{k_{h1}, k_{h2}\} < k < k_2$. So, when $0 < \alpha < \frac{1-d}{2}$, we let $k_h = \max\{k_{h1}, k_{h2}\}$ in equation (W13).

We can summarize the above discussion into Lemma W1, where k_l and k_h are given by

$$k_l \equiv (v - vr)(1-\alpha)md, \tag{W16}$$

$$k_h \equiv \begin{cases} (v - vr)md & \text{if } \frac{1-d}{2} \leq \alpha < \frac{1}{2}, \\ \max\{k_{h1}, k_{h2}\} & \text{if } 0 < \alpha < \frac{1-d}{2}. \end{cases} \tag{W17}$$

$\square$

With respect to d. Similar to the above analysis for k, we can derive the following thresholds:

$$d_h \equiv \frac{k}{(1-\alpha)(v-vr)m}, \tag{W18}$$

$$d_l \equiv \begin{cases} \frac{k}{(v-vr)m} & \text{if } \frac{1}{2}(1 - \frac{k}{(v-vr)m}) \leq \alpha < \frac{1}{2}, \\ \max\{d_{l1}, d_{l2}\} & \text{if } 0 < \alpha < \frac{1}{2}(1 - \frac{k}{(v-vr)m}). \end{cases} \tag{W19}$$

Note that k_l and d_h are defined by the same equation, and similarly for k_h and d_l. So, we have the following equivalence: $k \leq k_l$ is equivalent to $d \geq d_h$, and $k \geq k_h$ is equivalent to

$d \leq d_l$.

Part (2): with respect to α. We revisit the three regions discussed above. In particular, we only need to revisit Region (iii): $(1-\alpha)(v-vr)md = k_1 < k < k_2 = (v-vr)md$. Note that $k > k_1$ can be equivalently written as $\alpha > \alpha_l = \frac{(v-vr)md-k}{(v-vr)md}$.

Recall from equation (W13) that in this region, the platform chooses $x^* = 1$ if and only if $\max\{k_{h1}, k_{h2}\} < k < k_2$ and $\alpha < \frac{1-d}{2}$. Below, we will show that these conditions can be equivalently written as $\alpha_m < \alpha < \alpha_h$, where $\alpha_l < \alpha_m \leq \alpha_h < \frac{1-d}{2}$. This implies that, in Region (iii), the platform chooses $x^* < 1$ when $\alpha_l < \alpha < \alpha_m \cup \alpha_h < \alpha < \frac{1}{2}$.

Specifically, first, we will show that $k_{h1} < k < k_2$ can be translated into $\alpha_1 < \alpha < \alpha_2$, where $\alpha_l < \alpha_1 \leq \alpha_2 < \frac{1-d}{2}$; second, we will show that $k_{h2} < k < k_2$ is equivalent to $\alpha_3 < \alpha < \alpha_4$, where $\alpha_l < \alpha_3 \leq \alpha_4 \leq \frac{1-d}{2}$. Combining these two steps, $\max\{k_{h1}, k_{h2}\} < k < k_2$ can be equivalently translated into $\alpha_m < \alpha < \alpha_h$, where $\alpha_l < \alpha_m = \max\{\alpha_1, \alpha_3\} \leq \alpha_h = \min\{\alpha_2, \alpha_4\} < \frac{1-d}{2}$.

Prove that $k_{h1} < k < k_2$ can be translated into $\alpha_1 < \alpha < \alpha_2$, where $\alpha_l < \alpha_1 \leq \alpha_2 < \frac{1-d}{2}$.

First, one can show that $k > k_{h1} \Leftrightarrow g_1(\alpha) > 0$, where $g_1(\alpha)$ is a cubic function of α: $g_1(\alpha) = 8\alpha^3 d(d+1)m(1-r)v + 2\alpha^2\left(2\left(d^2+2d+2\right)k + d(7d+8)m(r-1)v\right) - \alpha\left(\left(4d^2+6d+8\right)k + d\left(d^2+7d+10\right)m(r-1)v\right) + (2+d+d^2)(k+dm(r-1)v)$. We note three observations regarding $g_1(\alpha)$: (i) the leading coefficient of α^3 is positive, implying that $g_1(\alpha) \to -\infty$ as $\alpha \to -\infty$ and $g_1(\alpha) \to \infty$ as $\alpha \to \infty$; (ii) $g_1(\alpha_l) = \frac{4k^2(dm(r-1)v+k)}{m^2(r-1)^2v^2} < 0$; (iii) $g_1(\frac{1-d}{2}) = d^2(d+1)^2(dm(r-1)v+k) < 0$. Thus, the region of α satisfying $g_1(\alpha) > 0$ on $\alpha_l < \alpha < \frac{1-d}{2}$ must be a (possibly empty) interval. That is, there exist α_1, α_2 satisfying $\alpha_l < \alpha_1 \leq \alpha_2 < \frac{1-d}{2}$ such that $g_1(\alpha) > 0$ if and only if $\alpha \in (\alpha_1, \alpha_2)$. The interval may be empty, in which case $\alpha_1 = \alpha_2$.

Prove that $k_{h2} < k < k_2$ can be translated into $\alpha_3 < \alpha < \alpha_4$, where $\alpha_l < \alpha_3 \leq \alpha_4 \leq \frac{1-d}{2}$.

Note that in Part (1), we compare $\Pi_P(x_2)$ with $\Pi_P(1)$ to derive k_{h2}. We note that the sign of $\Pi_P(x_2) - \Pi_P(1)$ depends on $g_2(\alpha) = 4\alpha^3 d^2(d+1)m^2(r-1)^2v^2 - \alpha^2 d^2 m(r-1)v((d+4)k+(5d+2)m(r-1)v) + \alpha\left(-d\left(d^2+4d+8\right)km(r-1)v - 4d^2m^2(r-1)^2v^2 - (d+2)^2k^2\right) + (d+2)(dm(r-1)v+k)^2$, which is a cubic function of α. We note three observations regarding $g_2(\alpha)$: (i) the leading coefficient of α^3 is positive, implying that $g_2(\alpha) \to -\infty$ as $\alpha \to -\infty$ and $g_2(\alpha) \to \infty$ as $\alpha \to \infty$; (ii) $g_2(\alpha_l) = \frac{2(d+2)k(dm(r-1)v+k)^2}{m(1-r)v} > 0$; (iii) $g_2(\frac{1-d}{2})$ can be positive or negative. Thus, there exist α_3, α_4 satisfying $\alpha_l < \alpha_3 \leq \alpha_4 \leq \frac{1-d}{2}$ such that $g_2(\alpha) < 0$ if and only if $\alpha \in (\alpha_3, \alpha_4)$. The interval may be empty, in which case $\alpha_3 = \alpha_4$. □

Proof of Proposition 3: Equilibrium Welfare Impacts When $\lambda = 1$

The proof follows the above analysis with $\lambda = 1$. The platform's profit function is given in equation (W10). Table W4 summarizes the equilibrium consumer surplus in the pricing subgame (i.e., given the sellers' external advertising decisions and the platform's product discoverability choice). Table W2 gives the sellers' equilibrium profits in the pricing subgame. Taking into account the optimal discoverability level x^* and the resulting external advertising decisions, we derive the *equilibrium* platform profit, seller total profit, and consumer surplus for the full game.

Table W4: Equilibrium consumer surplus in the pricing subgame

Seller advertising	Consumer Surplus
$I_A = I_B = N$	$v(2xm\alpha)$
$I_A = I_B = Y$	$v(2+2d)xm\alpha$
$I_j = Y, I_{-j} = N$	$v\frac{(1-x)m+(2+d)xm}{(1+d)(1-x)m+(2+d)xm}(2+d)xm\alpha$

We first prove the effects of k on the equilibrium outcomes: the platform's equilibrium profit Π_P^*, the sellers' total equilibrium profit $\Pi_A^* + \Pi_B^*$, and the equilibrium consumer surplus CS^*. Then, we further show the welfare effects of d and α.

(1) Welfare effects of k.

(1.1) As k increases, Π_P^* weakly decreases.

When $k \leq k_1$ or $k \geq k_2$, Π_P^* does not depend on k. When $k_1 < k < k_2$, the platform's equilibrium profit is chosen from feasible candidates including $\Pi_P(x_1)$, $\Pi_P(x_2)$, and $\Pi_P(1)$. As shown in the previous proof, $\Pi_P(x_1)$ and $\Pi_P(x_2)$ decrease with k (since x_1 and x_2 decrease with k), whereas $\Pi_P(1)$ does not vary with k. Therefore, Π_P^* weakly decreases with k on (k_1, k_2). At $k = k_2$, the YN/YY regions vanish and Π_P^* may drop discontinuously to $\Pi_P(1)$ when $\alpha > \frac{1-d}{2}$ (since $\Pi_P(x_1)|_{k=k_2} > \Pi_P(1)$ in this case, as shown above); otherwise the transition is continuous.

For parts (1.2) and (1.3) below, we focus on $\alpha < \frac{1-d}{2}$. Moreover, the proof below considers $\frac{1}{2}(\sqrt{17}-3) \leq d < 1$, under which $k_{h1} > k_{h2}$ and therefore $k_h = \max\{k_{h1}, k_{h2}\} = k_{h1}$. One can verify that all the results also hold when $0 < d < \frac{1}{2}(\sqrt{17}-3)$. Our goal is to provide examples supporting the proposition's existence results.

(1.2) As k increases, $\Pi_A^* + \Pi_B^*$ increases discontinuously at k_h.

In the case where $k_h = \max\{k_{h1}, k_{h2}\} = k_{h1}$, when $k \to k_{h1}^+$, $x^* = 1$ and neither seller invests; when $k \to k_{h1}^-$, $x^* = x_1$ and both sellers invest. Then, one can show that $\lim\limits_{k \to k_{h1}^+} \Pi_A^* + \Pi_B^* - \lim\limits_{k \to k_{h1}^-} \Pi_A^* + \Pi_B^* = \frac{2(\alpha-1)dm(r-1)v\left(8\alpha^2 d+8\alpha^2-6\alpha d-8\alpha+d^2+d+2\right)}{(2\alpha-1)(2\alpha d^2+4\alpha d+4\alpha-d^2-d-2)}$, which is positive given $\alpha < \frac{1-d}{2}$.

(1.3) As k increases, CS^* increases discontinuously at k_h.

In the case where $k_h = \max\{k_{h1}, k_{h2}\} = k_{h1}$, when $k \to k_{h1}^+$, $x^* = 1$ and neither seller invests; when $k \to k_{h1}^-$, $x^* = x_1$ and both sellers invest. Then, one can show that $\lim\limits_{k \to k_{h1}^+} CS^* - \lim\limits_{k \to k_{h1}^-} CS^* = \frac{2\alpha dmv}{1-2\alpha}$, which is positive given $\alpha < \frac{1-d}{2}$.

(2) Welfare effects of d. First, one can verify that the platform's equilibrium profit Π_P^* weakly increases with d. Also, as we have shown, the conditions for $k \lesseqgtr k_h = \max\{k_{h1}, k_{h2}\}$ can be equivalently expressed as $d \gtreqless d_l = \max\{d_{l1}, d_{l2}\}$. Therefore, if consumer sur-

plus/seller total profit *discontinuously* increases (decreases) when k increases over k_h, then it also *discontinuously* increases (decreases) when d decreases over d_l. This leads to the results in Proposition 3 regarding d.

(3) Welfare effects of α. First, one can verify that the platform's equilibrium profit Π_P^* decreases with α. Similar to above, if consumer surplus/seller total profit *discontinuously* increases (decreases) when k increases over k_h, then it also *discontinuously* increases (decreases) when α increases over α_m and when α decreases over α_h. This leads to the results in Proposition 3 regarding α. □

Detailed Characterization of "At Least One Invests" When $\lambda = 1$

Recall from Proposition 2 that when $k_l < k < k_h$ and $\alpha < 1/2$, the platform limits the discoverability to $x^* < 1$, and *at least one seller* invests. In this parameter region, either *only one seller invests* or *both sellers invest* in equilibrium. We now characterize the exact conditions for each scenario to occur.

Specifically, we show the following claim: *When $\alpha < 1/2$, there exist $k_l < \check{k}_l \leq \check{k}_h \leq k_h$ such that when $k \in (\check{k}_l, \check{k}_h)$, the platform chooses $x^* = x_2 < 1$, and only one seller invests in equilibrium. By contrast, when $k \in (k_l, \check{k}_l] \cup [\check{k}_h, k_h)$, the platform chooses $x^* = x_1 < 1$, and both sellers invest in equilibrium. Note that it is possible that $\check{k}_l = \check{k}_h$, and the platform chooses $x^* = x_1 < 1$ for all $k \in (k_l, k_h)$.*

We divide the discussion into two cases: $\frac{1-d}{2} \leq \alpha < \frac{1}{2}$ and $0 < \alpha < \frac{1-d}{2}$. First, we consider $k_l < k < k_h$ and $\frac{1-d}{2} \leq \alpha < \frac{1}{2}$. In this region, one can verify that the platform chooses $x^* = x_1 < 1$, and both sellers invest in equilibrium.

Second, we consider $k_l < k < k_h$ and $0 < \alpha < \frac{1-d}{2}$. In the proof of Proposition 2, we have shown that the platform chooses either $x^* = x_1$ or $x^* = x_2$ when $k_l < k < k_h$ and $0 < \alpha < \frac{1-d}{2}$. To pin down the platform's choice between $x^* = x_1$ and $x^* = x_2$, we compare

$\Pi_P(x_1)$ with $\Pi_P(x_2)$. We know that both $\Pi_P(x_1)$ and $\Pi_P(x_2)$ decrease with $k \in (k_l, k_h)$. In addition, one can show that (1) $\Pi_P(x_1)$ is convex regarding $k \in (k_l, k_h)$, (2) $\Pi_P(x_2)$ is concave regarding $k \in (k_l, k_h)$, and (3) $\Pi_P(x_1) - \Pi_P(x_2)|_{k=k_l} = 2(1-\alpha)vrmd > 0$. Since $\Pi_P(x_1) - \Pi_P(x_2)$ is convex in k (as the difference of a convex and a concave function) and strictly positive at $k = k_l$ by property (3), it has at most two zeros on (k_l, k_h), yielding at most one interior interval where $\Pi_P(x_1) < \Pi_P(x_2)$, which establishes the claimed structure.

As an example, Figure W6 illustrates the platform's optimal discoverability and the resulting external advertising. Figure W7 further shows that the equilibrium discoverability x^* decreases discretely at $\check{k}_l$ and $\check{k}_h$ when the region shifts from "only one seller invests" to "both sellers invest." This is consistent with our general insight: the platform needs to further limit discoverability to induce more sellers to invest in external advertising.

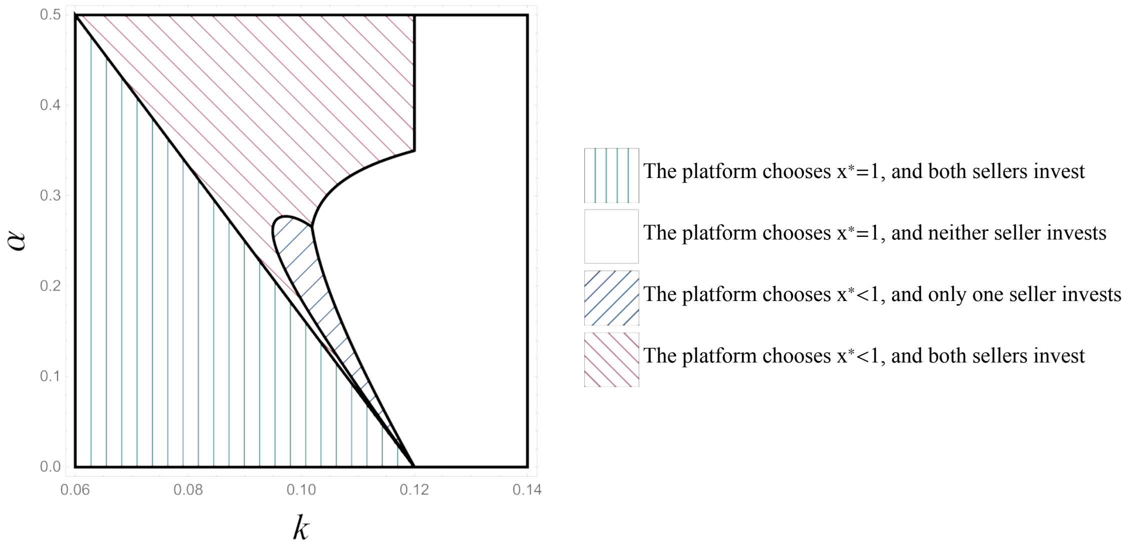


Figure W6: Equilibrium product discoverability and seller external advertising

Note. $v = 2$, $r = 0.5$, $m = 0.4$, $d = 0.3$, and $\lambda = 1$.

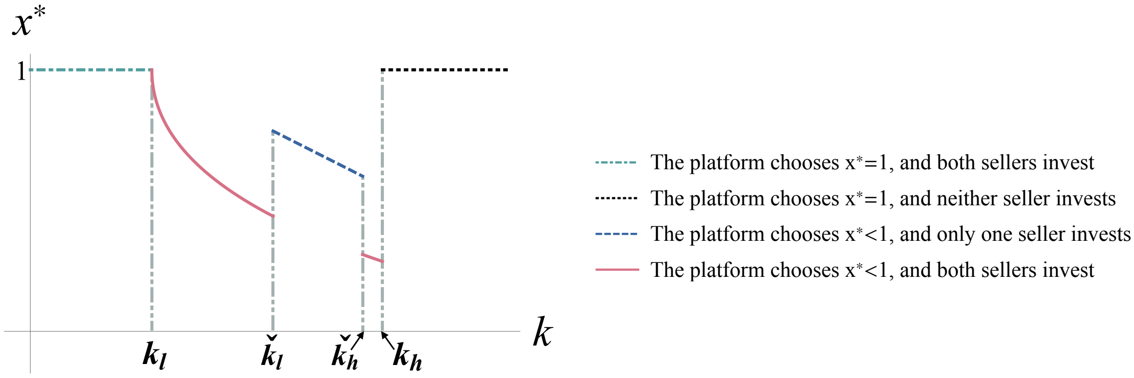


Figure W7: Effect of k on equilibrium discoverability

Note. $v = 2$, $r = 0.5$, $m = 0.4$, $d = 0.3$, $\alpha = 0.27$, and $\lambda = 1$.

Proof of Proposition 4: Optimal Discoverability When $\lambda = 0$

When $\lambda = 0$, $k_1 = 0$ and $k_2 = (1-\alpha)(v - vr)md$ in the proof of Lemma 1. In this case, our model setup requires $d < 1$ for regularity. The platform sets the discoverability level $x \in [0,1]$ to maximize its total profit:

$$\Pi_P(x) = \begin{cases} 2vmr(1+(1-2\alpha)x) & \text{if } k \geq k_2\text{, or if } 0 < k < k_2 \text{ and } x > x_2, \\ \frac{vmr(2(1+x)+(1-x)d)(1+(1-2\alpha)x+d(1-\alpha-x+\alpha x))}{1+d+x-dx} & \text{if } 0 < k < k_2 \text{ and } x_1 < x \leq x_2, \\ 2vmr(1+(1-2\alpha)x+(1-2\alpha)(1-x)d) & \text{if } 0 < k < k_2 \text{ and } x \leq x_1. \end{cases} \tag{W20}$$

The proof takes three steps. Note that we only need to focus on $0 < k < k_2$, because otherwise neither seller advertises for $x \in [0,1]$. First, we show that if the platform does not induce any seller to invest in external advertising, then it will set $x = 1$. Second, we show that the platform will never limit $x < 1$ to induce both sellers to invest in external advertising. Lastly, we show that if product substitutability is sufficiently large ($\alpha > \alpha_0$), the platform will limit product discoverability to $x < 1$ such that only one seller invests in external advertising.

First, if the platform does not induce any seller to invest in external advertising, then its profit will be $2vmr(1+(1-2\alpha)x)$, where $x_2 < x \leq 1$. In this case, it is straightforward that the platform will choose $x = 1$, leading to a profit of $\Pi_P(1) = 4(1-\alpha)vmr$. Below, we discuss whether the platform will earn a higher profit than $\Pi_P(1)$ by limiting $x \leq x_2$ to induce seller external advertising.

Second, if the platform induces both sellers to invest in external advertising, then its profit will be $\Pi_P^{YY}(x) = 2vmr(1+(1-2\alpha)x+(1-2\alpha)(1-x)d)$, where $0 \leq x \leq x_1$. One can readily show that $\Pi_P(1) > \Pi_P^{YY}(x)$ for all $0 \leq x \leq x_1$ since $x_1 < 1$ and $d < 1$. Therefore, the platform will never limit $x < 1$ to induce both sellers to invest in external advertising.

Lastly, if the platform induces only one seller to invest in external advertising, then its profit will be $\Pi_P^{YN}(x) = \frac{vmr(2(1+x)+(1-x)d)(1+(1-2\alpha)x+d(1-\alpha-x+\alpha x))}{1+d+x-dx}$, where $x_1 < x \leq x_2$. One can show that $\Pi_P(1) < \Pi_P^{YN}(x)$ if and only if $\alpha > \frac{(2-d)(1+d+x-dx)}{4+2d-d^2+(2-d)^2x}$. One can further show that $\frac{(2-d)(1+d+x-dx)}{4+2d-d^2+(2-d)^2x}$ decreases with x. Note that $0 \leq x_1 < x \leq x_2 < 1$, so $\Pi_P(1) < \Pi_P^{YN}(x)$ must hold when $\alpha > \frac{(2-d)(d(1-x)+x+1)}{4(x+1)+d(2-4x)-d^2(1-x)}|_{x=0} = \frac{(2-d)(1+d)}{4+2d-d^2} \equiv \alpha_0$. Therefore, if $\alpha > \alpha_0$, the platform limits product discoverability to $x^* < 1$ such that one seller invests in external advertising whereas the other seller does not. Note that $\alpha > \alpha_0$ is a sufficient condition for the platform to optimally induce only one seller to invest in external advertising. One can verify that $\alpha_0 < \frac{1}{2}$ for $d \in (0,1)$, so $\alpha > \alpha_0$ is feasible within the assumed parameter space $\alpha \in \left(0, \frac{1}{2}\right)$. □

Proof of Corollary 1: Non-Monotonic Platform Profit When $\lambda = 0$

We show that when $\alpha > \alpha_0$, the platform's equilibrium profit first increases and then decreases as k increases. By the proof of Proposition 4, when $\alpha > \alpha_0$ and $0 < k < k_2$, the YY regime is dominated and the YN regime strictly dominates NN: $\Pi_P^{YN}(x) > \Pi_P^{NN}(1)$ for all $x \in (0,1)$. Thus, the platform's equilibrium profit is $\Pi_P^*(k) = \Pi_P^{YN}(x^*(k))$, where $x^*(k)$ maximizes $\Pi_P^{YN}(x)$ over the feasible YN region $[x_1(k), x_2(k)]$, with x_1 defined by $\widehat{k}(x_1) = k$ (or $x_1 := 0$ when $k > k_{12}$, as in the proof of Lemma 1) and x_2 defined by $\overline{k}(x_2) = k$ (equivalently, $x_2 = 1 - k/k_2$). For $k \geq k_2$, neither seller advertises regardless of x, and the platform sets $x^* = 1$ with profit $\Pi_P^*(k) = \Pi_P^{NN}(1) = 4(1-\alpha)vmr$, which is constant in k.

Step 1: $\Pi_P^{YN}(x)$ *has a unique interior maximizer* $\widetilde{x} \in (0,1)$*, independent of* k*.*

From equation (W20), $\Pi_P^{YN}(x) = vmr \cdot A(x)B(x)/D(x)$ where $A(x) = (2+d)+(2-d)x$, $B(x) = 1+(1-2\alpha)x+d(1-\alpha)(1-x)$, and $D(x) = (1+d)+(1-d)x$. One can verify that $\frac{d^2}{dx^2}\left[\frac{A(x)B(x)}{D(x)}\right] = \frac{-8\alpha d}{D(x)^3} < 0$, so $\Pi_P^{YN}(x)$ is strictly concave in x. Evaluating the first order derivative at the boundaries: $\frac{d}{dx}\left[\frac{A(x)B(x)}{D(x)}\right]\Bigg|_{x=0} = \frac{(2+3d-d^3)-\alpha(4+4d+d^2-d^3)}{(1+d)^2}$, which is positive for all $\alpha < \frac{1}{2}$ and $d \in (0,1)$; and $\frac{d}{dx}\left[\frac{A(x)B(x)}{D(x)}\right]\Big|_{x=1} = (2-d) - \alpha(4-d)$, which is negative when $\alpha > \frac{2-d}{4-d}$. Since $\alpha_0 > \frac{2-d}{4-d}$ for $d \in (0,1)$, the condition $\alpha > \alpha_0$ ensures the derivative is positive at $x = 0$ and negative at $x = 1$, so the maximum is attained in the interior. Therefore, there exists a unique interior maximizer $\widetilde{x} \in (0,1)$, which depends on α and d but not on k.

Step 2: The equilibrium profit first increases and then decreases in k.

Since $\widehat{k}(x)$ and $\overline{k}(x)$ are strictly decreasing in x (as shown in the proof of Lemma 1), both $x_1(k)$ and $x_2(k)$ are decreasing in k. Moreover, $x_1, x_2 \to 1$ as $k \to 0^+$, and $x_2 \to 0$ as $k \to k_2^-$. Since $\widetilde{x} \in (0,1)$, there exist $0 < k_a < k_b < k_2$ such that $x_1(k_a) = \widetilde{x}$ and $x_2(k_b) = \widetilde{x}$ (with $k_a < k_b$ because $x_1 < x_2$ for $k \in (0, k_2)$).

For $k < k_a$: $x_1(k) > \widetilde{x}$, so the unconstrained maximizer lies below the feasible region $[x_1, x_2]$. The platform optimally sets $x^* = x_1$. By the chain rule, $\frac{d\Pi_P^*}{dk} = \Pi_P^{YN\prime}(x_1) \cdot x_1'(k)$. Since $x_1 > \widetilde{x}$, Π_P^{YN} is decreasing at x_1, so $\Pi_P^{YN\prime}(x_1) < 0$. Since x_1 is decreasing in k, $x_1'(k) < 0$. The product is positive, so profit increases.

For $k_a \leq k \leq k_b$: $\widetilde{x}$ is feasible, so $x^* = \widetilde{x}$ and the profit is constant at $\Pi_P^{YN}(\widetilde{x})$.

For $k > k_b$: $x_2(k) < \widetilde{x}$, so the unconstrained maximizer lies above the feasible region. The platform sets $x^* = x_2(k)$. By the chain rule, $\frac{d\Pi_P^*}{dk} = \Pi_P^{YN\prime}(x_2) \cdot x_2'(k)$. Since $x_2 < \widetilde{x}$, Π_P^{YN} is increasing at x_2, so $\Pi_P^{YN\prime}(x_2) > 0$. Since $x_2'(k) = -1/k_2 < 0$, the product is negative, so profit decreases. □

Numerical Illustration: Equilibrium Outcomes for General $\lambda \in (0,1)$

This subsection numerically explores equilibrium outcomes for general $\lambda \in (0,1)$, complementing our analyses of the two polar cases $\lambda = 1$ and $\lambda = 0$. Parts 1 and 2 examine two interior values, $\lambda = 0.9$ and $\lambda = 0.1$, to show that the key qualitative insights from the polar cases continue to hold when a seller's external advertising reaches a mix of consumers unaware of any product and consumers already aware of the competing product. For each value, we illustrate how equilibrium discoverability, profits, and welfare vary with product substitutability α, advertising effectiveness d, and advertising cost k. Part 3 then maps the equilibrium regimes of product discoverability and seller advertising over (k, α) and (d, α) for a range of λ values from 0 to 1, giving a more complete picture of how the regimes evolve as λ varies between the two polar cases. Together, the three parts show that the qualitative insights from the two polar cases can carry over to general λ.

Part 1: Equilibrium outcomes when $\lambda = 0.9$. In this case, a seller's external advertising substantially expands platform traffic, but it can also reach some consumers who already know the competing product.

Figure W8 shows how equilibrium outcomes vary with product substitutability α. Consistent with Proposition 1, the platform limits discoverability to induce both sellers to advertise when α is moderately low or very high, and sets full discoverability for intermediate α (where neither seller advertises). The counterintuitive welfare implications in Proposition 3(i) also carry over: as α increases, total seller profit can increase and consumer surplus can decrease, despite stronger competition between sellers.

Figures W9 and W10 show how equilibrium outcomes vary with the cost-effectiveness of external advertising, captured by d and k respectively. Consistent with Proposition 2, the platform limits discoverability to induce both sellers to advertise when cost-effectiveness is in a moderate range, and sets full discoverability otherwise. The counterintuitive welfare implications in Proposition 3(ii) also carry over: under higher advertising cost-effectiveness, both total seller profit and consumer surplus can decrease.

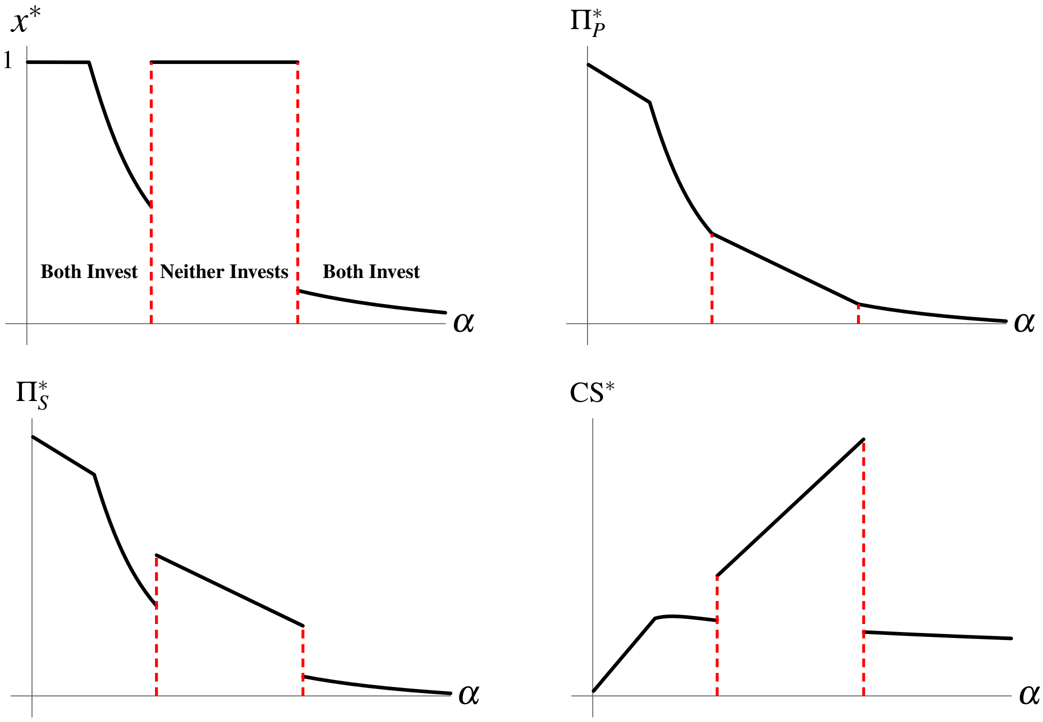


Figure W8: Effects of α on equilibrium outcomes under $\lambda = 0.9$

Note. $v = 2$, $r = 0.5$, $m = 0.4$, $d = 0.3$, and $k = 0.1$.

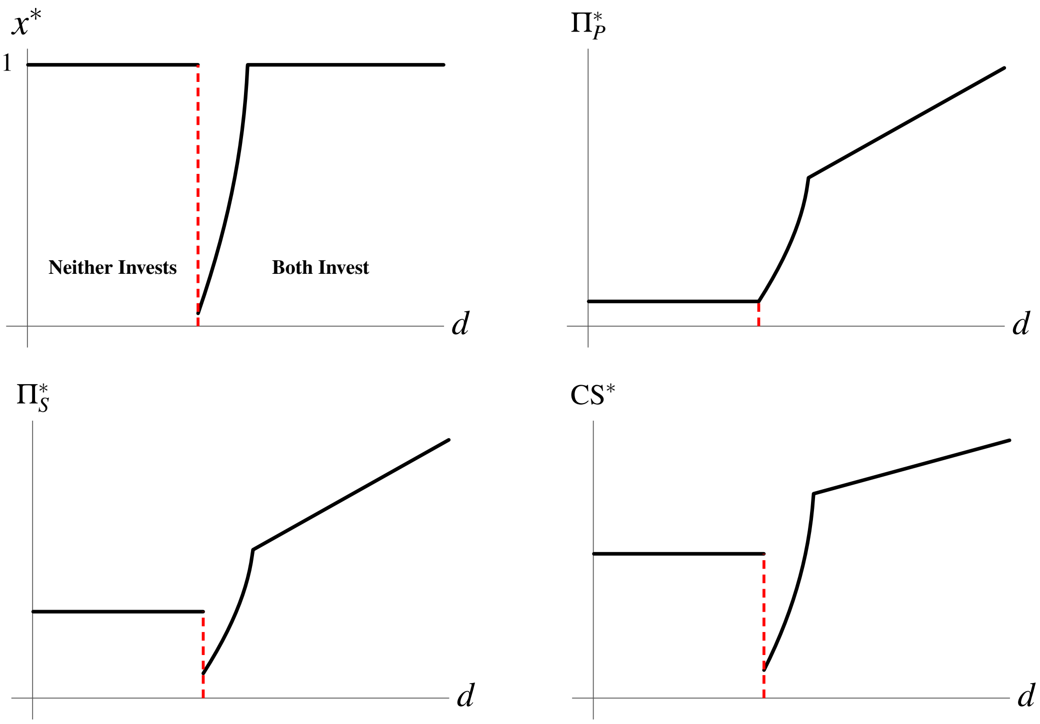


Figure W9: Effects of d on equilibrium outcomes under $\lambda = 0.9$

Note. $v = 2$, $r = 0.5$, $m = 0.4$, $\alpha = 0.3$, and $k = 0.1$.

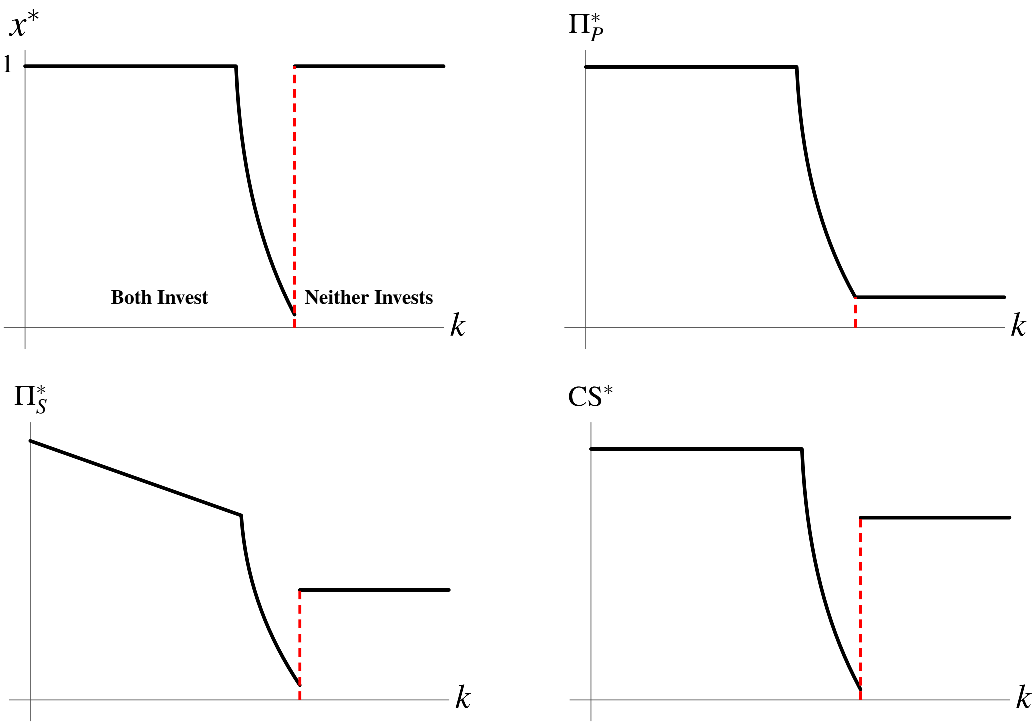


Figure W10: Effects of k on equilibrium outcomes under $\lambda = 0.9$

Note. $v = 2$, $r = 0.5$, $m = 0.4$, $d = 0.3$, and $\alpha = 0.3$.

Part 2: Equilibrium outcomes when $\lambda = 0.1$. In this case, a seller's external advertising primarily reaches consumers who already know the competing product, but it can also bring some new consumers to the platform.

Figure W11 shows how equilibrium outcomes vary with product substitutability α. Consistent with Proposition 4, the platform sets full discoverability with no seller advertising when α is low, and switches to limiting discoverability to induce only one seller to advertise once α becomes sufficiently large.

Figure W12 further shows how equilibrium outcomes vary with advertising cost k. As k increases from low to high, the equilibrium transitions from both sellers advertising, to only one seller advertising, and finally to neither seller advertising. Unlike the $\lambda = 0$ polar case where external advertising brings no new traffic, here external advertising still attracts some new consumers to the platform, so it retains a positive traffic-expansion effect. At low k, the platform would only need to lower discoverability slightly to induce asymmetric advertising, and the seller differentiation benefit is therefore limited. As a result, the platform chooses to induce both sellers to advertise rather than just one. As k rises into the moderate range, the platform must lower discoverability further to incentivize advertising, and the seller differentiation benefit becomes large enough that inducing only one seller to advertise dominates. The platform's equilibrium profit then exhibits a non-monotonic pattern because of the differentiation mechanism, as in Corollary 1.

Figure W13 shows how equilibrium outcomes vary with advertising effectiveness d. As d increases, the equilibrium transitions from neither seller advertising, to only one seller advertising, and then to both sellers advertising. The intuition is similar to the above.

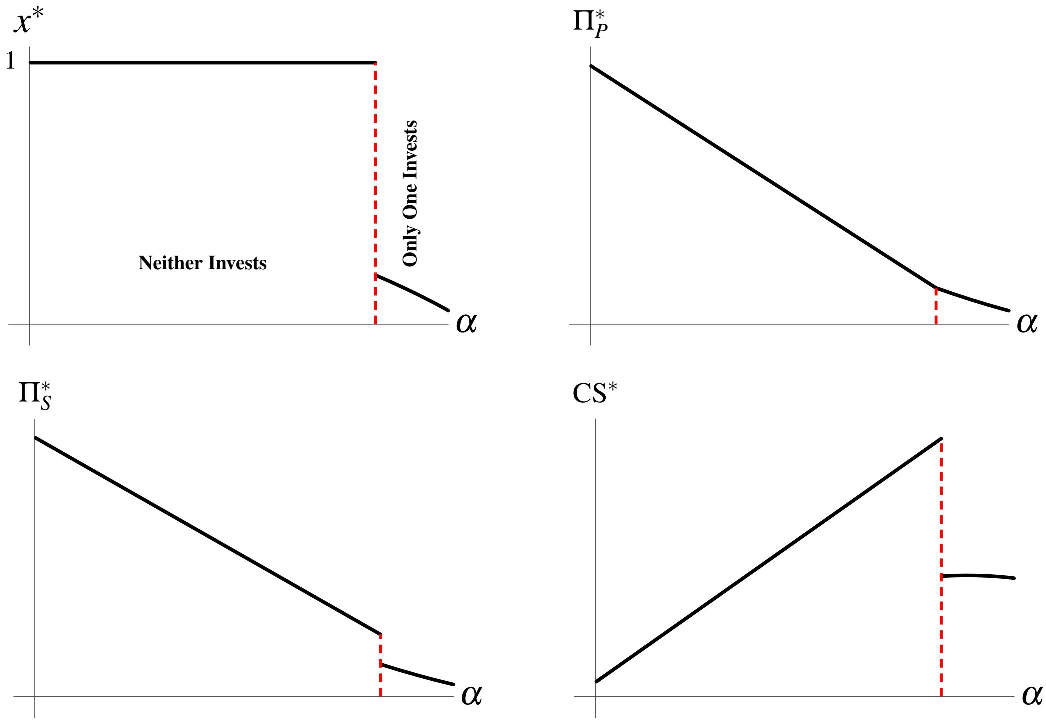


Figure W11: Effects of α on equilibrium outcomes under $\lambda = 0.1$

Note. $v = 2$, $r = 0.5$, $m = 0.4$, $d = 0.6$, and $k = 0.1$.

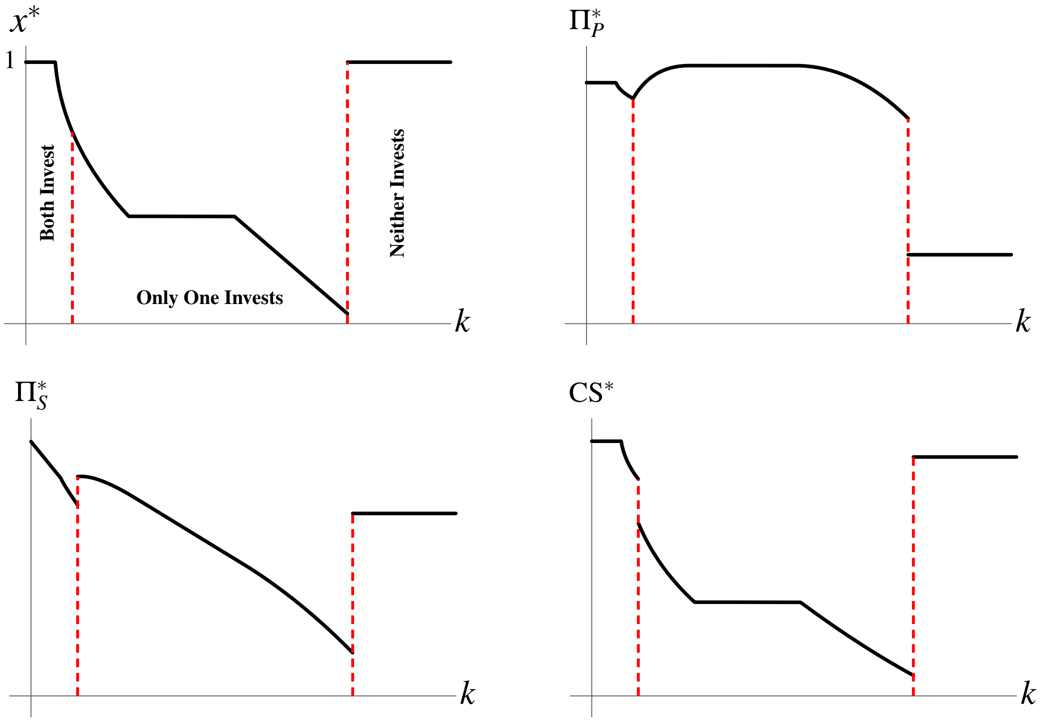


Figure W12: Effects of k on equilibrium outcomes under $\lambda = 0.1$

Note. $v = 2$, $r = 0.5$, $m = 0.4$, $d = 0.6$, and $\alpha = 0.48$.

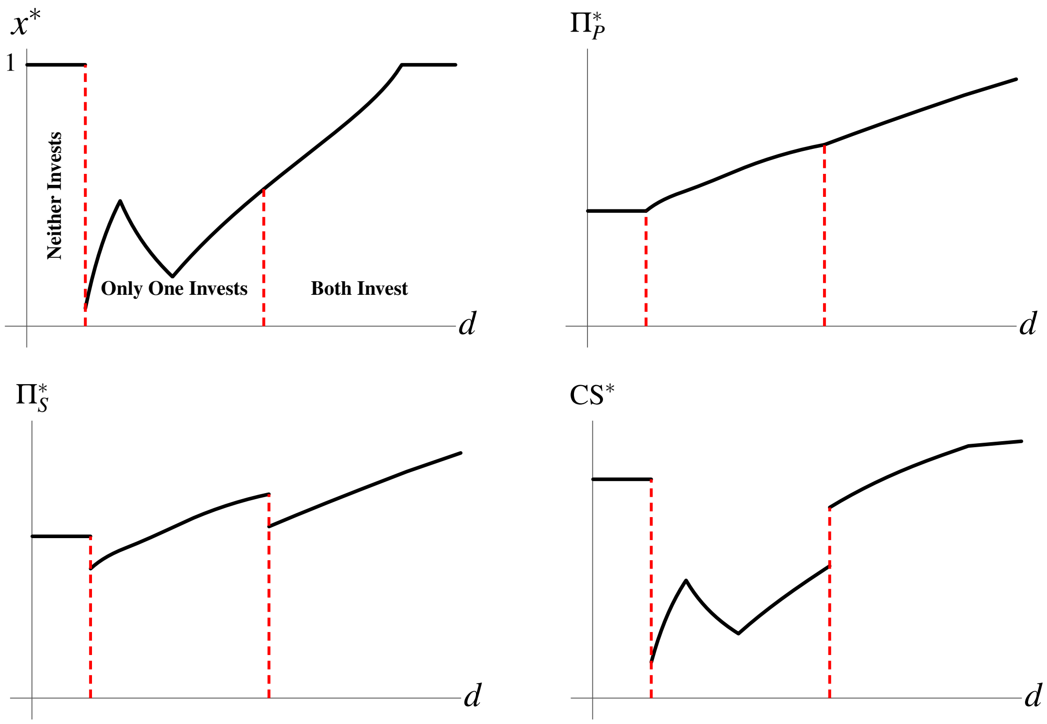


Figure W13: Effects of d on equilibrium outcomes under $\lambda = 0.1$

Note. $v = 2$, $r = 0.5$, $m = 0.4$, $\alpha = 0.48$, and $k = 0.02$.

Part 3: Equilibrium regimes of product discoverability and seller advertising for different λ values. Figures W14 and W15 map the equilibrium regime of product discoverability x^* and seller advertising decisions over (k, α) and (d, α), respectively, for a range of λ values from 0 to 1 (with other parameter values held fixed). Each region indicates both which sellers advertise (none, one, or both) and whether the platform sets full discoverability ($x^* = 1$) or limits it ($x^* < 1$). Recall that a larger λ means external advertising brings more new consumers and reaches fewer consumers who already know the competing seller.

"Both Invest with $x^* = 1$" region. This region is empty at $\lambda = 0$ and expands as λ increases. When λ is larger, a seller's return from advertising strengthens, so sellers are more willing to advertise even at full discoverability.

"Both Invest with $x^* < 1$" region. This region is also empty at $\lambda = 0$, first emerges and expands as λ rises above 0, then eventually shrinks as λ grows large. To benefit from traffic expansion, the platform needs to lower discoverability to incentivize both sellers to advertise. When λ is large, sellers become willing to advertise even at higher values of x^*, so limiting discoverability is less needed.

"Only One Invests with $x^* < 1$" region. This region exists even at $\lambda = 0$, reflecting the differentiation mechanism we discussed. It expands slightly as λ rises above 0 and then shrinks as λ grows large. When λ is small but positive, inducing only one seller to advertise not only yields the seller differentiation benefit but also brings a few new consumers, making it even more attractive. When λ is large, inducing both sellers to advertise delivers a substantial traffic-expansion benefit to the platform.

"Neither Invests with $x^* = 1$" region. This region shrinks as λ increases. Because external advertising brings more new consumers, sellers are more willing to advertise, and the platform is also more willing to induce advertising for traffic expansion.

Overall, for general $\lambda \in (0,1)$, our key qualitative insights are robust. First, the platform's optimal discoverability is non-monotonic in the cost-effectiveness of external advertising (captured by lower k or higher d): the platform sets $x^* = 1$ when cost-effectiveness is very low (where neither seller advertises) and when it is very high (where both sellers advertise even at full discoverability), and limits discoverability to $x^* < 1$ in the intermediate range. Second, the platform's optimal discoverability can also be non-monotonic in product substitutability α: the platform tends to set $x^* = 1$ for intermediate α and limit discoverability for low or high α.

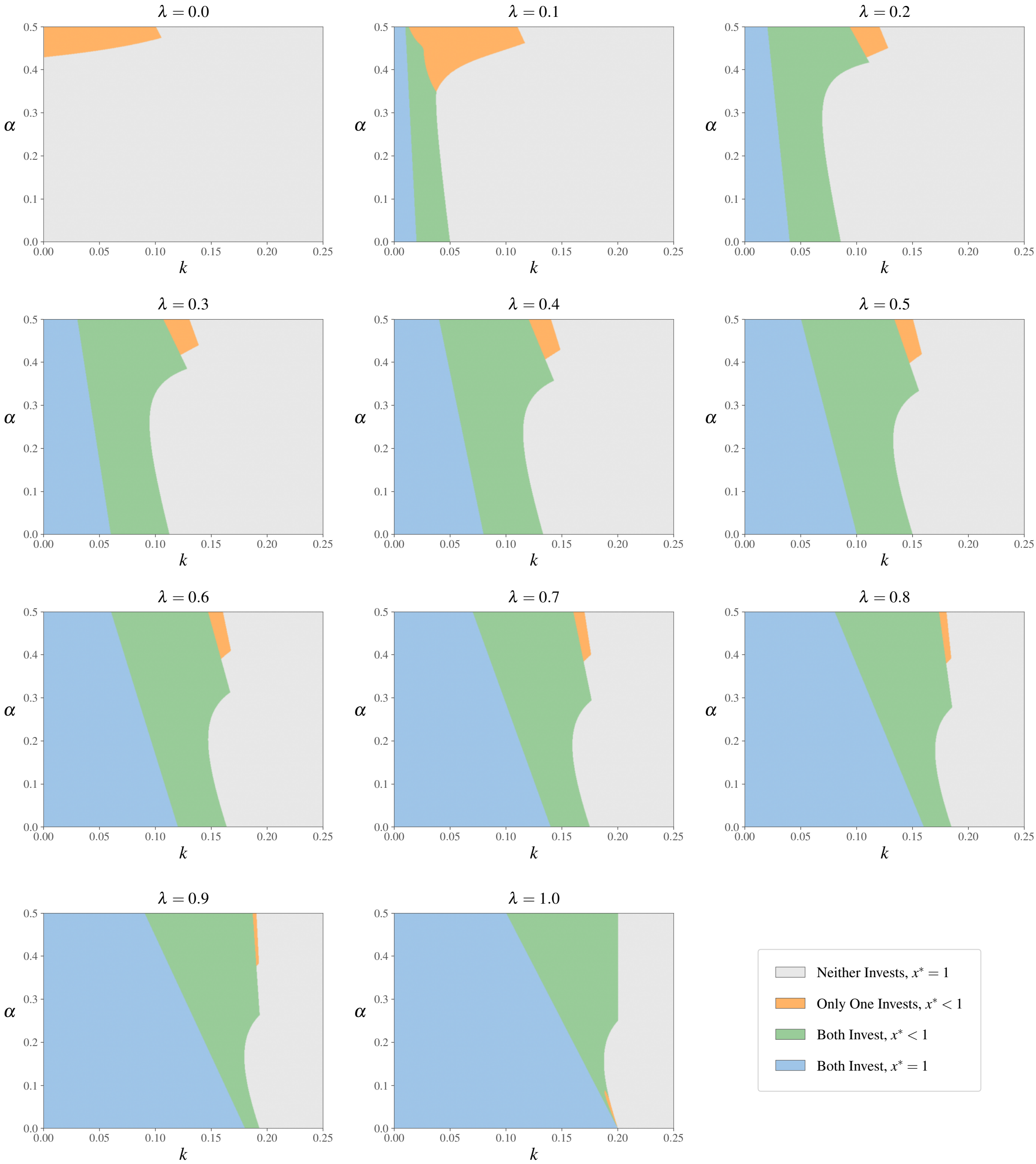


Figure W14: Equilibrium regime over (k, α) for different λ values

Note. $v = 2$, $r = 0.5$, $m = 0.4$, and $d = 0.5$.

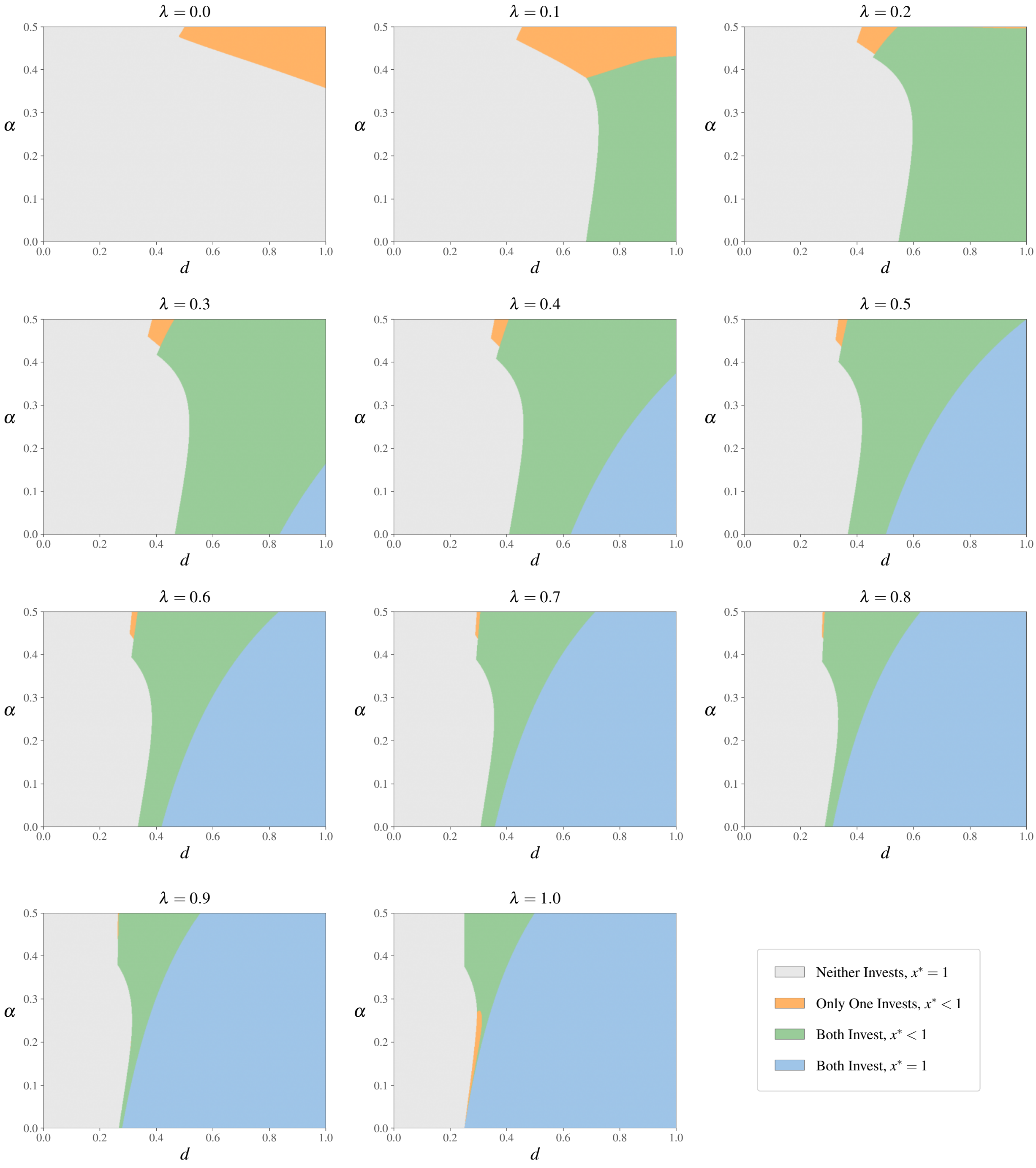


Figure W15: Equilibrium regime over (d, α) for different λ values

Note. $v = 2$, $r = 0.5$, $m = 0.4$, and $k = 0.1$.

PROOFS AND ANALYSIS FOR MODEL EXTENSIONS

Optimal Commission Under Fixed Product Discoverability

We prove the following lemma for the case with fixed product discoverability x. We consider $\lambda = 1$. Note that the platform can set the commission rate arbitrarily close to one, i.e., $r = 1 - \epsilon$ for an arbitrarily small $\epsilon > 0$, to leave sellers a positive margin and thus preserve our previous pricing equilibrium characterization. For notational convenience, the following analysis admits the limit $r \to 1$ (as $\epsilon \downarrow 0$), with the understanding that we essentially consider $r \leq 1-\epsilon$ for an arbitrarily small $\epsilon > 0$. This entails no loss of generality: whenever the supremum of platform profit is approached as $r \uparrow 1$, the continuity of $\Pi_P(r)$ within each regime ensures that $\Pi_P(1-\epsilon)$ converges to $\Pi_P(1)$, so all qualitative profit comparisons are preserved for sufficiently small ϵ. When instead the optimal commission is strictly interior (i.e., $r^* < 1$), it is exactly achievable.

Lemma W2 (Optimal commission under fixed product discoverability)**.** *Given a fixed product discoverability level x: (i) If seller external advertising has low cost-effectiveness (when $d < \widetilde{d}$ or equivalently $k > \widetilde{k}$), the platform sets $r^* = 1$, and neither seller invests in external advertising. (ii) If seller external advertising has high cost-effectiveness (when $d \geq \widetilde{d}$, or equivalently when $k \leq \widetilde{k}$), the platform sets $r^* < 1$, and at least one seller invests in external advertising.*

Proof: The proof takes two steps. In step (1), we characterize the equilibrium of the seller external advertising subgame with respect to the commission rate r, based on the proof of Lemma 1. In step (2), we derive the platform's optimal commission given discoverability x.

Step (1): we characterize the subgame equilibrium of seller external advertising with r.

We can rewrite $k < \widehat{k}(x)$ in the proof of Lemma 1 as $r < \widehat{r}_+(x) \equiv \max\{0, \widehat{r}(x)\}$, where

$$\widehat{r}(x) = 1 - \frac{k(d+x+1)}{\Big((d+1)(1-\alpha(2-x)x) + x - \alpha x\Big)vmd}. \tag{W21}$$

Thus, when $r < \widehat{r}_+(x)$, we have $\Pi_A^{YY} > \Pi_A^{NY}$ and $\Pi_B^{YY} > \Pi_B^{YN}$.

Similarly, we can rewrite $k < \overline{k}(x)$ in the proof of Lemma 1 as $r < \overline{r}_+(x) \equiv \max\{0, \overline{r}(x)\}$, where

$$\overline{r}(x) = 1 - \frac{k}{(1-\alpha x)vmd}. \tag{W22}$$

Thus, when $r < \overline{r}_+(x)$, we have $\Pi_A^{YN} > \Pi_A^{NN}$ and $\Pi_B^{NY} > \Pi_B^{NN}$.

Moreover, it holds that (i) both $\overline{r}(x)$ and $\widehat{r}(x)$ decrease with x; (ii) $\overline{r}(x) \geq \widehat{r}(x)$, where the equality is achieved at $x = 0$ or $x = 1$; (iii) $\overline{r}(0) = \widehat{r}(0) = 1 - \frac{k}{vmd}$; (iv) $\overline{r}(1) = \widehat{r}(1) = 1 - \frac{k}{(1-\alpha)vmd}$.

Therefore, YY is an equilibrium if and only if $r \leq \widehat{r}_+(x)$, NN is an equilibrium if and only if $r \geq \overline{r}_+(x)$, and YN or NY is an equilibrium if and only if $\widehat{r}_+(x) \leq r \leq \overline{r}_+(x)$.

Step (2): we derive the platform's optimal commission r^*.

The platform chooses the commission rate $r \in [0,1]$ to maximize its total profit:

$$\Pi_P(r) = \begin{cases} 2vmr(1+(1-2\alpha)x) & \text{if } \overline{r}_+(x) < r \leq 1, \\ \frac{(2+d)vmr(1+x)(1+(1-2\alpha)x+d(1-\alpha x))}{1+d+x} & \text{if } \widehat{r}_+(x) < r \leq \overline{r}_+(x), \\ (2+2d)vmr(1+(1-2\alpha)x) & \text{if } 0 < r \leq \widehat{r}_+(x), \end{cases} \tag{W23}$$

We discuss three cases with $\widehat{k}_+ = \frac{((d+1)(1-\alpha(2-x)x)+x-\alpha x)vmd}{d+x+1}$ and $\overline{k}_+ = (1-\alpha x)vmd$.

Case (i): $k \geq \overline{k}_+$. In this case, $\overline{r}_+(x) = \widehat{r}_+(x) = 0$. Therefore, the platform chooses $r^* = 1$, and neither seller invests in external advertising.

Case (ii): $\widehat{k}_+ \leq k < \overline{k}_+$. In this case, $\overline{r}_+(x) = \overline{r}(x) > 0 = \widehat{r}_+(x)$. Then, the platform's profit maximization problem is reduced to choosing r among $\{1, \overline{r}(x)\}$. If the platform chooses $r = 1$, neither seller invests; if the platform chooses $r = \overline{r}(x)$, only one seller invests. Thus, we compare $\Pi_P(1)$ with $\Pi_P(\overline{r})$. One can show $\Pi_P(1) < \Pi_P(\overline{r}) \Leftrightarrow k < \widetilde{k}_1 = \frac{d^2 mv(1-\alpha x)\left(d(x+1)(1-\alpha x)+(1-4\alpha)x^2+2x+1\right)}{(d+2)(x+1)(d(1-\alpha x)+(1-2\alpha)x+1)}$.

Case (iii): $k < \widehat{k}_+$. In this case, $\overline{r}_+(x) \geq \widehat{r}_+(x) > 0$. Then, the platform's profit maximization problem is reduced to choosing r among $\{1, \overline{r}(x), \widehat{r}(x)\}$. If the platform chooses $r = 1$, neither seller invests; if the platform chooses $r = \overline{r}(x)$ or $r = \widehat{r}(x)$, at least one seller invests. Therefore, the platform sets $r^* < 1$ if and only if $\Pi_P(1) < \max\{\Pi_P(\overline{r}), \Pi_P(\widehat{r})\}$. We have discussed $\Pi_P(1) < \Pi_P(\overline{r})$. One can further show $\Pi_P(1) < \Pi_P(\widehat{r}) \Leftrightarrow k < \widetilde{k}_2 = \frac{d^2 mv\left(\alpha d(x-2)x+\alpha x^2-3\alpha x+d+x+1\right)}{(d+1)(d+x+1)}$. Thus, $\Pi_P(1) < \max\{\Pi_P(\overline{r}), \Pi_P(\widehat{r})\}$ when $k < \max\{\widetilde{k}_1, \widetilde{k}_2\}$.

Let $\widetilde{k} \equiv \max\{\widetilde{k}_1, \widetilde{k}_2\}$. Then, when $k > \widetilde{k}$, the platform chooses $r^* = 1$, and neither seller invests in external advertising; when $k \leq \widetilde{k}$, the platform chooses $r^* < 1$, and at least one seller invests. One can also translate the results into d, as $\widetilde{k}$ increases with d. $\square$

Proof of Proposition 5: Optimal Discoverability and Commission

Below, we derive a *sufficient* condition such that the platform will choose $x^* < 1$ and $r^* < 1$ in equilibrium. We focus on the region of $k < (1-\alpha)vmd$; there may be other parameter regions with $k \geq (1-\alpha)vmd$ in which the platform will also set $x^* < 1$ and $r^* < 1$ in equilibrium. As noted, the proof considers $\lambda = 1$. The ϵ-limit convention for $r \to 1$ introduced in the proof of Lemma W2 applies throughout.

When $0 < k < (1-\alpha)vmd$, we have $1 > \overline{r}(x) \geq \widehat{r}(x) > 0$, $\forall x \in [0,1]$. The platform chooses discoverability $x \in [0,1]$ and commission $r \in [0,1]$ to maximize its profit

$$\Pi_P(x,r) = \begin{cases} 2vmr(1+(1-2\alpha)x) & \text{if } \overline{r}(x) < r \leq 1 \text{ and } 0 \leq x \leq 1, \\ \frac{(2+d)vmr(1+x)(1+(1-2\alpha)x+d(1-\alpha x))}{1+d+x} & \text{if } \widehat{r}(x) < r \leq \overline{r}(x) \text{ and } 0 \leq x \leq 1, \\ (2+2d)vmr(1+(1-2\alpha)x) & \text{if } 0 < r \leq \widehat{r}(x) \text{ and } 0 \leq x \leq 1, \end{cases} \tag{W24}$$

For any fixed x, $\Pi_P(x,r)$ is increasing in r within each feasible region. Therefore, the optimal commission must be attained at the upper boundary of the relevant region, that is, the platform's optimal commission rate must be among $\{\widehat{r}(x), \overline{r}(x), 1\}$. Hence the platform's optimization problem can be reduced as $\max\{\Pi_{P1}, \Pi_{P2}, \Pi_{P3}\}$, where

$$\Pi_{P1} \equiv \max_{x\in[0,1]} H_1(x) = \max_{x\in[0,1]} 1 \cdot 2vm(1+(1-2\alpha)x), \tag{W25}$$

$$\Pi_{P2} \equiv \max_{x\in[0,1]} H_2(x) = \max_{x\in[0,1]} \overline{r}(x) \cdot \frac{(2+d)vm(1+x)(1+(1-2\alpha)x+d(1-\alpha x))}{1+d+x}, \tag{W26}$$

$$\Pi_{P3} \equiv \max_{x\in[0,1]} H_3(x) = \max_{x\in[0,1]} \widehat{r}(x) \cdot (2+2d)vm(1+(1-2\alpha)x). \tag{W27}$$

In other words, we can first find the optimal x for $H_1(x)$, $H_2(x)$, and $H_3(x)$, respectively; then, we can compare Π_{P1}, Π_{P2}, and Π_{P3} to locate the optimal x and r for the ultimate maximization problem $\max_{x\in[0,1], r\in[0,1]} \Pi_P(x,r)$. In particular, if Π_{P2} or Π_{P3} is larger than Π_{P1}, it means that the optimal r is $\overline{r}(x)$ or $\widehat{r}(x)$, thereby $r^* < 1$.

Clearly, $\operatorname{argmax}_{x\in[0,1]} H_1(x) = 1$ and $\Pi_{P1} = vm(4-4\alpha)$. The sufficient condition for Proposition 5 is comprised of the following three conditions: (1) $\operatorname{argmax}_{x\in[0,1]} H_2(x) < 1$, (2) $\operatorname{argmax}_{x\in[0,1]} H_3(x) < 1$, and (3) $\max\{\Pi_{P2}, \Pi_{P3}\} > \Pi_{P1}$. Together, they imply that the platform chooses $x^* < 1$ and $r^* < 1$. We derive the three conditions in order.

First, we find a condition such that $\operatorname{argmax}_{x\in[0,1]} H_2(x) < 1$. Specifically, we show that the derivative of $H_2(x)$ at $x = 1$ is negative when $k > \frac{dmv(1-\alpha)(d+2-\alpha(3d+4))}{(1-\alpha)d+2}$, which implies that the maximizer of $H_2(x)$ must be smaller than 1.

Second, we find a condition such that $\operatorname{argmax}_{x\in[0,1]} H_3(x) < 1$. Specifically, we show that $H_3(0) > H_3(1)$ when $k > (1-2\alpha)dmv$, which implies that the maximizer of $H_3(x)$ must be smaller than 1.

Third, we find a condition such that $\max\{\Pi_{P2}, \Pi_{P3}\} > \Pi_{P1}$. Specifically, we show that $H_3(0) > \Pi_{P1}$ when $k < \frac{dmv(2\alpha+d-1)}{d+1}$, which implies $\max\{\Pi_{P2}, \Pi_{P3}\} \geq \Pi_{P3} \geq H_3(0) > \Pi_{P1}$.

Putting together, the platform chooses $x^* < 1$ and $r^* < 1$ when $\widetilde{k}_l < k < \widetilde{k}_h$, where

$$\widetilde{k}_l \equiv \max\left\{(1-2\alpha)dmv, \frac{dmv(1-\alpha)(d+2-\alpha(3d+4))}{(1-\alpha)d+2}\right\}, \tag{W28}$$

$$\widetilde{k}_h \equiv \min\left\{(1-\alpha)dmv, \frac{dmv(2\alpha+d-1)}{d+1}\right\}. \tag{W29}$$

We can also translate $\widetilde{k}_l < k < \widetilde{k}_h$ into $\widetilde{d}_h > d > \widetilde{d}_l$. □

Proof of Both $x^* < 1$ and $r^* < 1$ Under $\lambda = 0$

As in the $\lambda = 1$ case, we derive a *sufficient* condition such that the platform will choose $x^* < 1$ and $r^* < 1$ in equilibrium. As in Proposition 4, we assume $d \in (0,1)$ for regularity. The ϵ-limit convention for $r \to 1$ from the proof of Lemma W2 applies throughout.

Specializing the thresholds $\overline{k}(x)$ and $\widehat{k}(x)$ from the proof of Lemma 1 to $\lambda = 0$ and inverting in r yields

$$\overline{r}(x) = 1 - \frac{k}{(1-\alpha)(1-x)\,vmd}, \tag{W30}$$

$$\widehat{r}(x) = 1 - \frac{k\left[(1+d)+(1-d)x\right]}{vmd\left[(1+d)-\alpha(1+2d)+\Big(2\alpha(2d-1)-2d\Big)x+\Big(\alpha(3-2d)+d-1\Big)x^2\right]}. \tag{W31}$$

Note that $\overline{k}(1) = \widehat{k}(1) = 0$, so both $\overline{r}(x)$ and $\widehat{r}(x)$ tend to $-\infty$ as $x \to 1^-$. In particular, the YN and YY regimes are *infeasible at* $x = 1$: for any $k > 0$, no positive commission rate sustains them there.

For any (x, r), the platform's profit is:

$$\Pi_P(x,r) = \begin{cases} 2vmr\Big(1 + (1-2\alpha)x\Big) & \text{if } \overline{r}(x) < r \le 1, \\ \dfrac{vmr\left[(2+d) + (2-d)x\right]\left[1 + (1-2\alpha)x + d(1-\alpha)(1-x)\right]}{(1+d) + (1-d)x} & \text{if } \widehat{r}(x) < r \le \overline{r}(x), \\ 2vmr\Big[1 + (1-2\alpha)\Big((1-d)x + d\Big)\Big] & \text{if } 0 < r \le \widehat{r}(x), \end{cases} \tag{W32}$$

where each branch is taken to be empty whenever the corresponding threshold is non-positive (as in the proof of Lemma W2). For any fixed x, $\Pi_P(x, r)$ is increasing in r within each feasible branch, so the optimal commission must lie in $\{\widehat{r}(x), \overline{r}(x), 1\}$. Hence the platform's joint optimization reduces to $\max\{\Pi_{P1}, \Pi_{P2}, \Pi_{P3}\}$, where

$$\Pi_{P1} \equiv \max_{x\in[0,1]} H_1(x) = \max_{x\in[0,1]} 2vm\Big(1 + (1-2\alpha)x\Big), \tag{W33}$$

$$\Pi_{P2} \equiv \max_{x\in[0,1]} H_2(x) = \max_{x\in[0,1]} \overline{r}(x) \cdot \frac{vm\Big[(2+d) + (2-d)x\Big]\Big[1 + (1-2\alpha)x + d(1-\alpha)(1-x)\Big]}{(1+d) + (1-d)x}, \tag{W34}$$

$$\Pi_{P3} \equiv \max_{x\in[0,1]} H_3(x) = \max_{x\in[0,1]} \widehat{r}(x) \cdot 2vm\Big[1 + (1-2\alpha)\Big((1-d)x + d\Big)\Big], \tag{W35}$$

with $H_2(x)$ (resp. $H_3(x)$) set to $-\infty$ wherever $\overline{r}(x) \le 0$ (resp. $\widehat{r}(x) \le 0$).

Clearly, $\operatorname{argmax}_{x\in[0,1]} H_1(x) = 1$ and $\Pi_{P1} = vm(4 - 4\alpha)$.

The sufficient condition we derive consists of two parts: (i) $\Pi_{P3} \le \Pi_{P1}$ (YY is dominated by NN), and (ii) $\Pi_{P2} > \Pi_{P1}$. Together they imply that the joint maximum equals Π_{P2}, attained in the YN regime. Since YN is infeasible at $x = 1$, the maximizer must satisfy $x^* < 1$, and the platform sets $r^* = \overline{r}(x^*) < 1$ (strict because $k > 0$).

Step 1 ($\Pi_{P3} \le \Pi_{P1}$). Since $\alpha < 1/2$ and $(1-d)x + d \le 1$ on $[0,1]$, the per-r YY profit in equation (W32) satisfies

$$2vm\Big[1 + (1-2\alpha)\Big((1-d)x + d\Big)\Big] \;\le\; 2vm\Big[1 + (1-2\alpha)\Big] \;=\; \Pi_{P1}.$$

Multiplying by $\widehat{r}(x) \le 1$ where it is positive gives $H_3(x) \le \Pi_{P1}$; outside the admissible region, $H_3 = -\infty \le \Pi_{P1}$ by convention. Hence $\Pi_{P3} = \max_{x\in[0,1]} H_3(x) \le \Pi_{P1}$.

Step 2 ($\Pi_{P2} > \Pi_{P1}$). It suffices to show $H_2(x_0) > \Pi_{P1}$ at a single $x_0 \in [0,1)$; we use $x_0 = 0$, where

$$\overline{r}(0) \;=\; 1 - \frac{k}{(1-\alpha)\,vmd}, \qquad H_2(0) \;=\; \overline{r}(0) \cdot \frac{vm\,(2+d)\,\Big[1 + d(1-\alpha)\Big]}{1+d}.$$

The strict inequality $H_2(0) > \Pi_{P1} = vm(4-4\alpha)$ rearranges to

$$k \;<\; \widetilde{k}_0 \;\equiv\; (1-\alpha)\,vmd\left[1 - \frac{4(1-\alpha)(1+d)}{(2+d)\Big(1+d(1-\alpha)\Big)}\right].$$

This $\widetilde{k}_0$ is positive if and only if the bracket is positive, which is equivalent to $\alpha > \alpha_0 = \frac{(2-d)(1+d)}{4+2d-d^2}$, the same threshold as in the proof of Proposition 4. The bracket is below 1, so $\widetilde{k}_0 < (1-\alpha)vmd$ and hence $\overline{r}(0) > 0$ at the anchor $x_0 = 0$.

Putting together, the platform chooses $x^* < 1$ and $r^* < 1$ when $k < \widetilde{k}_0$. $\square$

Some Platform Visitors Are Unaware of Any Product

In this extension, we assume that a size $\eta > 0$ of visitors arrive at the platform without knowing any seller's product. Product discoverability allows a fraction x of these unaware visitors to discover both products, and the rest do not discover any product. (The results are qualitatively the same if some of the latter discover only one product.) Other model settings remain unchanged. For simplicity, below we only present the analysis when external advertising fully expands platform traffic ($\lambda = 1$) and illustrate the robustness of the key insights in the main model. (Our key insights under $\lambda = 0$ can also hold qualitatively.)

Similar to the main model, we derive the equilibrium outcomes using backward induction. Both advertising cost-effectiveness and product substitutability can impact the optimal discoverability in a non-monotonic way. Paralleling the main model analysis, we find that there exist thresholds $k_{l\eta} < k_{h\eta}$ such that when $k \leq k_{l\eta}$, the platform sets $x^* = 1$ and both sellers invest in external advertising; when $k_{l\eta} < k < k_{h\eta}$, the platform sets $x^* < 1$ and at least one seller invests; when $k \geq k_{h\eta}$, the platform sets $x^* = 1$ and no seller invests, where

$$k_{l\eta} \equiv (v - vr)(1 - \alpha)md, \tag{W36}$$

$$k_{h\eta} \equiv \begin{cases} (v - vr)md & \text{if } \alpha \geq \frac{\eta+1-d}{\eta+2}, \\ \max\{k_{h1,\eta}, k_{h2,\eta}\} & \text{if } \alpha < \frac{\eta+1-d}{\eta+2}. \end{cases} \tag{W37}$$

Moreover, we find that as the mass of unaware visitors grows, the platform will have less incentive to limit discoverability to induce external traffic, that is, $k_{h\eta}$ decreases with η. So, our key insights are more relevant when η is not too large. That said, we also emphasize that even for sufficiently large η ($\alpha < \frac{\eta+1-d}{\eta+2} \Leftrightarrow \eta > \frac{2\alpha+d-1}{1-\alpha}$), the region where the platform limits $x^* < 1$ persists. This can be shown by $k_{h\eta} - k_{l\eta} \geq k_{h1,\eta} - k_{l\eta} > 0$.

Sellers' External Advertising Levels Are Continuous

In the main model, each seller's external advertising decision is binary: either pays the advertising cost to expand awareness or does not advertise. This extension allows advertising to be continuous. Specifically, seller j chooses an advertising intensity $y_j \in [0, 1]$ and incurs cost $k \cdot (y_j + y_j^2)$—so advertising has a baseline per-unit cost and becomes increasingly expensive to scale up. For tractability, we assume that a seller's ad succeeds with probability y_j, which yields high awareness $1 + d$, and fails with probability $1 - y_j$, which yields low awareness 1, independent across sellers. The d additional consumers reached by a successful ad are all new to the platform (reflecting $\lambda = 1$ in the main model where external advertising fully expands platform traffic; our key insights under $\lambda = 0$ can also hold qualitatively). Other model settings remain unchanged.

Consequently, there are four *ex post* awareness cases: both sellers have high awareness $(1+d, 1+d)$ with probability $y_A \cdot y_B$; both sellers have low awareness $(1, 1)$ with probability $(1-y_A)\cdot(1-y_B)$; seller A has high awareness whereas seller B has low awareness $(1+d, 1)$ with probability $y_A \cdot (1-y_B)$; seller A has low awareness whereas seller B has high awareness $(1, 1 + d)$ with probability $(1 - y_A) \cdot y_B$.

Similar to the main model, we derive the equilibrium using backward induction. In each ex-post case, the demand composition follows the main model analysis (under $\lambda = 1$), so do the demand function and the pricing equilibrium analysis. We derive the sellers' equilibrium profits in each ex-post case. For the advertising equilibrium, given y_{-j}, seller j decides its advertising intensity y_j, expecting the four ex-post cases and the advertising cost $k \cdot (y_j + y_j^2)$. Anticipating the sellers' pricing and advertising decisions, the platform maximizes its total expected commissions by choosing the discoverability level.

Below we provide a numerical analysis to illustrate the robustness of our core insights. We focus on symmetric advertising equilibrium where $y_A^* = y_B^* = y^*$. Figure W16 shows that when the cost parameter k is intermediate, the platform reduces the discoverability level $x^* < 1$ to incentivize seller external advertising; otherwise, the platform sets the highest possible level $x^* = 1$. Note that the sellers' equilibrium advertising intensity y_j^* may take interior values (between 0 and 1). Regarding welfare implications, when k increases, both sellers and consumers can be better off; this is because the platform strategically raises the discoverability level whereas the sellers' advertising intensity decreases.

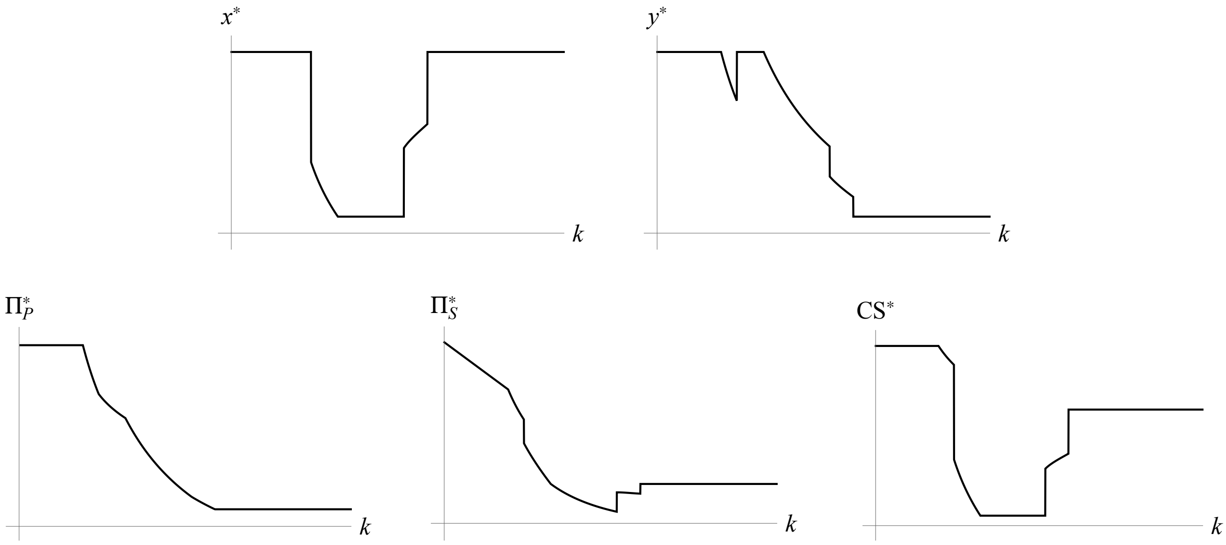


Figure W16: The effects of k under continuous advertising

Note. $v = 2$, $r = 0.5$, $m = 0.4$, $\alpha = 0.4$, and $d = 0.6$.

External Advertising Also Has a Persuasion Effect

This extension allows external advertising to also carry a persuasion effect that increases the advertised seller's product valuation for consumers. In particular, we assume that seller j's external advertising can increase its product matching probability m by $\gamma \geq 0$ for consumers reached by the advertising (hereafter, ad-exposed consumers; their size is d). The increased probability γ is an encroachment of the original probability that consumers do not match with seller j, i.e., $1-m$. Moreover, the encroachment is proportional to the original probability that consumers match only seller $-j$ and the original probability that consumers match neither seller. Therefore, the probability of ad-exposed consumers matching both sellers is $m\alpha + \gamma\frac{m(1-\alpha)}{1-m}$, the probability of ad-exposed consumers matching only the advertising seller j is $m(1-\alpha)+\gamma\frac{1-m(2-\alpha)}{1-m}$, the probability of ad-exposed consumers matching only the non-advertising seller $-j$ is $m(1-\alpha) - \gamma\frac{m(1-\alpha)}{1-m}$, and the probability of ad-exposed consumers matching neither seller is $1 - m(2-\alpha) - \gamma\frac{1-m(2-\alpha)}{1-m}$. Note that $\gamma < 1-m$. All the other settings remain unchanged as in the main model, which is a special case of $\gamma = 0$.

For simplicity, below we only present the analysis when external advertising fully expands platform traffic ($\lambda = 1$) and illustrate the robustness of the key insights in the main model. (Our key insights under $\lambda = 0$ can also hold qualitatively.) The equilibrium derivation is similar to the main model analysis using backward induction. Table W5 summarizes the demand composition under different seller advertising scenarios, including the impact of advertising persuasion γ. Figure W17 illustrates the robustness of our key insights in the presence of advertising persuasion: the platform may limit discoverability to motivate sellers' external advertising when advertising cost (and similarly effectiveness) is moderate, and when product substitutability is either moderately low or high.

Table W5: Consumer demand composition with advertising persuasion

Seller advertising	A-loyals (n_A)
$I_A = I_B = N$	$(1+x-2x\alpha)m$
$I_A = Y, I_B = N$	$(1+x-2x\alpha)m + d(1-x)(m+\gamma) + dx\left(m(1-\alpha) + \gamma\frac{1-(2-\alpha)m}{1-m}\right)$
$I_A = N, I_B = Y$	$(1+x-2x\alpha)m + dx\left(m(1-\alpha) - \gamma\frac{m(1-\alpha)}{1-m}\right)$
$I_A = I_B = Y$	$(1+x-2x\alpha)m + d(1-x)(m+\gamma) + dx\left(m(1-\alpha) + \gamma\frac{1-(2-\alpha)m}{1-m}\right) + dx\left(m(1-\alpha) - \gamma\frac{m(1-\alpha)}{1-m}\right)$

Seller advertising	B-loyals (n_B)
$I_A = I_B = N$	$(1+x-2x\alpha)m$
$I_A = Y, I_B = N$	$(1+x-2x\alpha)m + dx\left(m(1-\alpha) - \gamma\frac{m(1-\alpha)}{1-m}\right)$
$I_A = N, I_B = Y$	$(1+x-2x\alpha)m + d(1-x)(m+\gamma) + dx\left(m(1-\alpha) + \gamma\frac{1-(2-\alpha)m}{1-m}\right)$
$I_A = I_B = Y$	$(1+x-2x\alpha)m + d(1-x)(m+\gamma) + dx\left(m(1-\alpha) + \gamma\frac{1-(2-\alpha)m}{1-m}\right) + dx\left(m(1-\alpha) - \gamma\frac{m(1-\alpha)}{1-m}\right)$

Seller advertising	Switchers (n_{AB})	Total demand ($n_A + n_B + n_{AB}$)
$I_A = I_B = N$	$2x\alpha m$	$2m(1+x-\alpha x)$
$I_A = Y, I_B = N$	$2x\alpha m + dx\left(m\alpha + \gamma\frac{m(1-\alpha)}{1-m}\right)$	$(2+d)m(1+x-\alpha x) + \gamma\frac{d(1-m(1+x-\alpha x))}{1-m}$
$I_A = N, I_B = Y$	$2x\alpha m + dx\left(m\alpha + \gamma\frac{m(1-\alpha)}{1-m}\right)$	$(2+d)m(1+x-\alpha x) + \gamma\frac{d(1-m(1+x-\alpha x))}{1-m}$
$I_A = I_B = Y$	$2x\alpha m + 2dx\left(m\alpha + \gamma\frac{m(1-\alpha)}{1-m}\right)$	$2(1+d)m(1+x-\alpha x) + \gamma\frac{2d(1-m(1+x-\alpha x))}{1-m}$

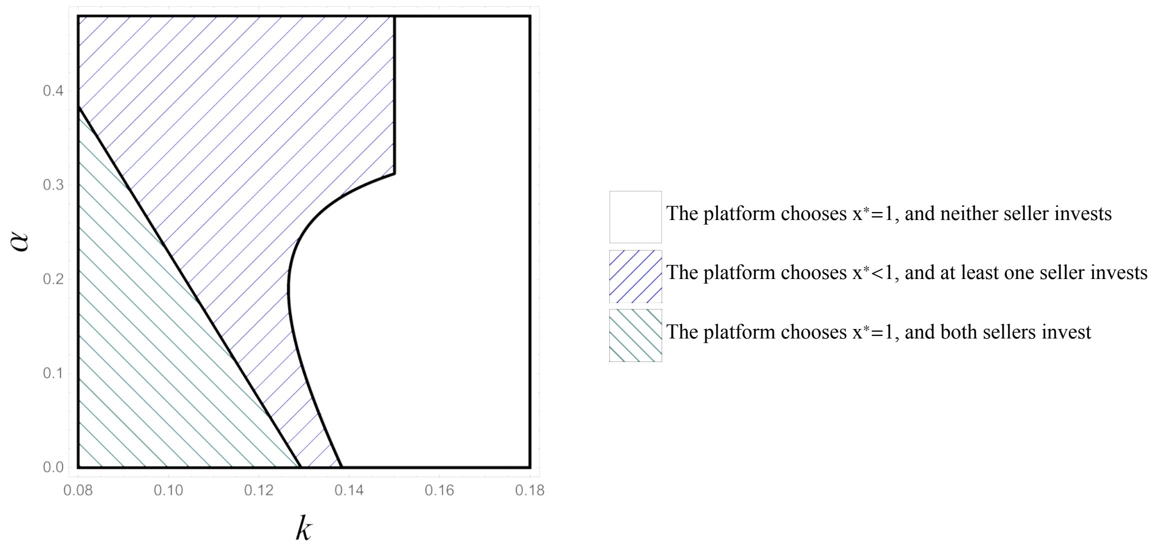


Figure W17: Equilibrium discoverability with advertising persuasion

Note. $v = 2$, $r = 0.5$, $m = 0.4$, $d = 0.3$, $\lambda = 1$, and $\gamma = 0.1$.

Consumers May Encounter Both Sellers' External Ads

In the main model, when both sellers advertise, their ad-reached consumers are disjoint. This extension allows for the possibility that some consumers encounter both sellers' ads. As in the main model, each seller's ad reaches a mass d of consumers previously unaware of its product. When only one seller advertises, the setup remains unchanged. When both sellers advertise: each seller's ad still reaches $(1-\lambda)d$ consumers who already know the competing product and λd consumers who initially know neither product. Among the latter, $\delta\lambda d$ encounter both sellers' ads and learn both products through external advertising, whereas $(1-\delta)\lambda d$ encounter only that seller's ad, where $\delta \in [0,1]$. The total traffic to the platform when both sellers advertise is $2+\lambda d(2-\delta)$. The main model is thus the special case $\delta = 0$. All the other settings remain unchanged as in the main model. Table W6 summarizes the consumer awareness composition in this extended model.

Table W6: Consumer awareness composition before visiting the platform

Seller advertising	Know only A (s_A)	Know only B (s_B)	Know both (s_{AB})	Total traffic ($s_A+s_B+s_{AB}$)
$I_A = I_B = N$	1	1	0	2
$I_A = Y, I_B = N$	$1+\lambda d$	$1-(1-\lambda)d$	$(1-\lambda)d$	$2+\lambda d$
$I_A = N, I_B = Y$	$1-(1-\lambda)d$	$1+\lambda d$	$(1-\lambda)d$	$2+\lambda d$
$I_A = I_B = Y$	$1-(1-\lambda)d+(1-\delta)\lambda d$	$1-(1-\lambda)d+(1-\delta)\lambda d$	$\delta\lambda d+2(1-\lambda)d$	$2+\lambda d(2-\delta)$

For simplicity, below we only present the analysis when external advertising fully expands platform traffic ($\lambda = 1$) and illustrate the robustness of the key insights in the main model. (Our key insights under $\lambda = 0$ can also hold qualitatively.) The equilibrium derivation is similar to the main model analysis using backward induction. Figure W18 illustrates the robustness of our key insight: the platform may limit discoverability to encourage sellers' external advertising. A new region arises though: only one seller advertises even at full discoverability ($x^* = 1$). This is because the possibility of seeing both ads allows more consumers to know and compare both products, which intensifies price competition and makes asymmetric advertising more appealing.

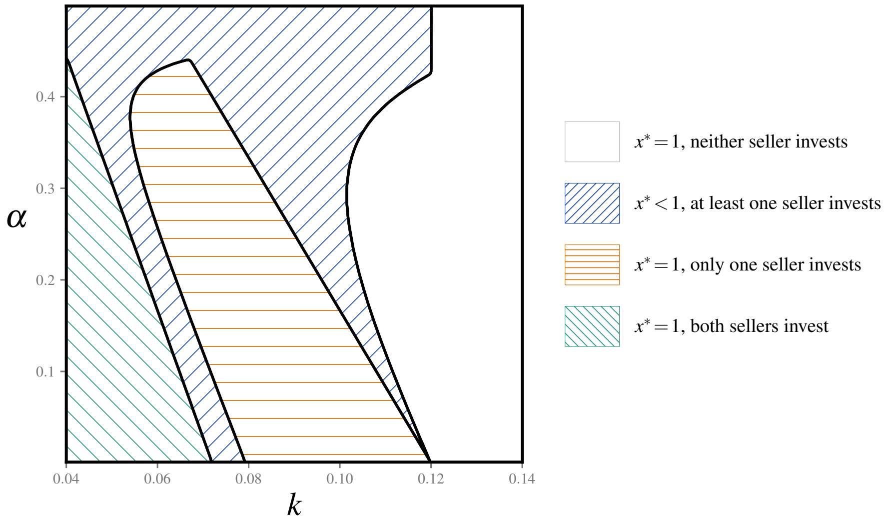


Figure W18: Equilibrium discoverability when consumers may encounter both sellers' ads

Note. $v = 2$, $r = 0.5$, $m = 0.4$, $d = 0.3$, $\lambda = 1$, and $\delta = 0.4$.